Mechanism Design meets Priority Design: Redesigning the US Army's Branching Process

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Winter School 2021, Delhi School of Economics
December 15-18, 2021

Army's Branching Process

- Each year, the US Army assigns thousands of graduating cadets from the United States Military Academy (USMA) at West Point and the Reserve Officer Training Corps (ROTC) to their first job in a military occupation, or branch, through centralized systems.
  - Branch assignment is highly consequential for career progression.
- Prior to the Class of 2006, cadets were assigned positions at Army branches using a serial dictatorship that is induced by a cadet performance ranking known as the order of merit list (OML).
  - Under this mechanism, cadets submit their preferences over the set of branches, and the highest-OML cadet is assigned her most-preferred branch, the second highest-OML cadet is assigned her most-preferred branch among branches with remaining positions, etc.

BRADSO Program and the 2006 Branching Reform

- In response to declining junior officer retention rates during the late 1990s and early 2000s, starting with 2006 the Army offered a menu of retention incentives to cadets at USMA and ROTC.
- The most popular incentive, which involved a reform of the branching mechanism, was the branch of choice (BRADSO) program.
  - Under this program, cadets are given higher priority for a fraction of positions at any given branch if they indicate willingness to extend their Active Duty Service Obligation (ADSO) by three years at that branch.
  - Terminology: We sometimes refer to ADSO as the price.
  - The strategy space of the new mechanism was also expanded by requesting cadets to report the set of branches for which they are willing to pay the increased price in exchange for receiving increased-priority at a fraction of its positions.

USMA-2006 Mechanism

- Under the USMA-2006 mechanism, the branch assignments are made through a process that resembles the previous OML-induced serial dictatorship, with one important exception:
  - Once the regular (base-price) positions are filled at any branch, cadets who indicated willingness to pay the increased price are given priority for the remaining flexible-price positions.
- The ADSO charges are determined subsequently as follows:
  - Cadets who receive a regular position are charged the regular price.
  - Cadets who receive a flexible-price position are charged
    - the regular price if they have not indicated willingness to pay the increased price for their assigned branch, and
    - the increased price if they have indicated willingness to pay the increased price for their assigned branch.

Shortcomings of the USMA-2006 Mechanism

- Two aspects of the USMA-2006 mechanism are not ideal:
  1. Cadets are asked to whether they are willing to pay the increased price at a branch or not independent of what the alternative is.

     For example, a cadet is not able to indicate

     - he is willing to pay the increased price to receive a position at his first choice branch if the alternative is receiving a position at his third or lower choice branches,
     - but not if the alternative is receiving a position at his second choice branch.
  2. Cadets who indicate willingness to pay the increased price for a branch are charged the increased price upon receiving one of its flexible-price positions even if they would have received the same position at a regular price in the absence of their willingness.

Shortcomings of the USMA-2006 Mechanism

- These aspects, in turn, result in a number of shortcomings of the USMA-2006 mechanism, including the following two:
  - Detectable Priority Reversal: A cadet may receive a position at increased price, while a lower-OML cadet receives a position at the same branch at regular price.
  - Failure of Incentive Compatibility: A cadet may benefit from hiding her willingness to pay the increased price (failure of BRADSO-IC) or from misrepresenting her branch preferences.

Initial Proposal of the Cumulative Offer Mechanism

- As a remedy, Sönmez & Switzer (2013) proposed an alternative mechanism for the USMA based on Hatfield & Milgrom's celebrated cumulative offer mechanism.
  - Proposal built on the fundamental research on matching with contracts by Hatfield & Milgrom (2005) and its very important extension by Hatfield & Kojima (2010).
- The proposed mechanism is a direct mechanism where cadets submit their preferences over branch-price pairs.
  - The Army initially viewed this strategy space to be too complex, and decided to maintain the USMA-2006 mechanism.

Army's Reasons to Maintain the USMA-2006 Mechanism

- Adoption of a mechanism with a more involved strategy space was initially seen at the Army as unnecessary due to three main reasons:
    1. BRADSO-IC failures and detectable priority reversals have been rare in practice.
    2. Any BRADSO-IC failure or detectable priority reversal can be manually corrected ex-post, since each incidence only involves a cadet needlessly paying the increased price at her assigned branch.
    3. While there can be additional priority reversals that cannot be manually corrected ex-post, their verification relies on cadet preferences over branch-price pairs, an information unavailable under the existing USMA-2006 strategy space.
- In summary, any failure of the USMA-2006 mechanism can either be manually corrected ex-post or cannot be verified with existing data.

Talent-Based Branching Program

- In 2012, the Army introduced a Talent-Based Branching (TBB) program to develop a "talent market" where additional information about each cadet influences the priority a cadet receives at a branch.
- Under the TBB program, branches rate cadets into one of three tiers: High, Medium, and Low.
  - For several years these ratings remained a pilot initiative.
  - For the Class of 2020, the Army decided to integrate them into the branching process, constructing priorities at each branch first by the tier and then by the OML within the tier.
  - BRADSO policy also changed: cadets willing to pay the increased price now received higher priority within their tier only.

USMA-2020 Mechanism

- Since the decision to integrate cadet ratings into branching process took place under an abbreviated timeline, the Army maintained the same strategy space for the new mechanism as in previous years.
- Using an adjusted priority order of cadets that takes both TBB ratings and increased-price willingness into consideration, the new mechanism used the cadet-proposing deferred acceptance algorithm (Gale & Shapley 1962) to determine the branch assignments.
- The ADSO charges were then determined subsequently as follows:
  - Subject to a maximum of the number of flexible-price positions at any given branch and following the reverse-priority order of the branch, cadets who indicated willingness to pay the increased price at their assigned branch are charged the increased price, and
  - the remaining cadets who are matched are charged the regular price.

Shortcomings of the USMA-2020 Mechanism

- In addition to inheriting the limitations of the USMA-2006 mechanism, the following aspect of the USMA-2020 mechanism added new challenges:
    - Even though the number of flexible-price positions was kept at 25% of the total capacity at each branch, priority upgrading due to increased-price willingness was applied for all its positions.
    - This design choice made it possible to use the vanilla version of the cadet-proposing deferred acceptance algorithm, but it also introduced a new type of incentive compatibility failure we call Strategic BRADSO.
    - Whereas indicating willingness to pay the increased price could hurt cadets due to BRADSO-IC failures, now it could also profit them with a costless priority upgrade due to Strategic BRADSO.
- The end result was a mechanism that is highly complex to participate in, and one with more widespread failures including priority reversals that cannot be manually corrected ex-post.

Concerns on the USMA-2020 Mechanism

- Before a formal analysis was carried out by our team, the USMA leadership already recognized the possibility of detectable priority reversals under the USMA-2020 mechanism due to either failure of BRADSO-IC or presence of strategic BRADSO.
- A major concern emerged as an erosion of cadets' trust in the Army's branching process due to failure of BRADSO-IC, possibility of strategic BRADSO, or presence of detectable priority reversals, especially when not manually corrected ex-post.
- To address these types of concerns, the USMA leadership decided to execute a dry run of the USMA-2020 mechanism to inform cadets of the potential cutoffs for each branch.

Concerns on the USMA-2020 Mechanism

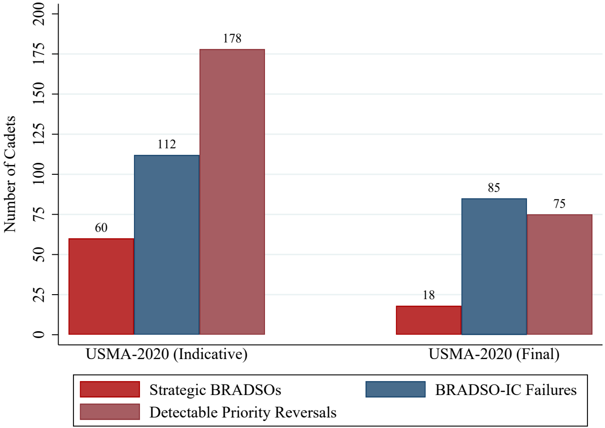
- As emphasized in the following quote from a September 2019 U.S. Army news article, the goal of the dry run was to improve transparency and help cadets to optimize their submitted strategies:

  "We're going to tell all the cadets, we're going to show all of them, here's when the branch would have went out, here's the bucket you're in, here's the branch you would have received if this were for real. You have six days to go ahead and redo your preferences and look at if you want to BRADSO or not." Sunsdahl said. "I think it's good to be transparent. I just don't know what 21-year-olds will do with that information."

- The same quote, however, also indicates that USMA leadership recognized the challenges in cadets optimizing their strategies under the USMA-2020 mechanism.

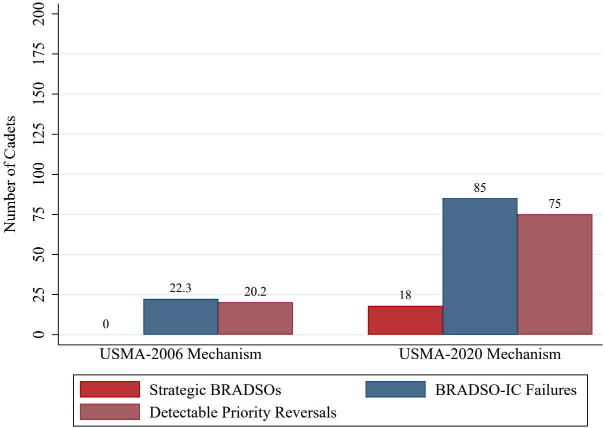
Failure Prevalence under Dry vs. Actual USMA-2020 Runs

- The left side correspond to the number of failures under the indicative strategies submitted in a dry-run and the right side correspond to the number of failures under the actual run of the USMA-2020 mechanism for the Class of 2020.



Problems Aggravated Compared to USMA-2006

- The left side correspond to the average number of failures of the USMA-2006 mechanism for Classes of 2014-2019, and the right side correspond to the number of failures under the USMA-2020 Mechanism for the Class of 2020.



Army's Partnership with Market Designers

- To resolve the problems with the branching system, the Army established a partnership with Pathak and Sönmez for a redesign of their mechanism, with Greenberg leading the reform efforts at USMA.
- Critical to achieving these objectives was the Army's decision to permit cadets in the Class of 2021 to submit preferences over branch-price pairs.
- This decision was aided by evidence from a cadet survey that mitigated concerns that ranking branch-price pairs would be overly complex or unnecessary.

Army's Partnership with Market Designers

- Indeed, some of the cadets indicated the need for a system that would allow them to rank order branch-price pairs. One cadet wrote:

  "[...] I believe that DMI (Department of Military Instruction) could elicit a new type of ranking list. Within my proposed system, people could add to the list of 17 branches BRADSO slots and rank them within that list. For example: AV (Aviation) > IN (Infantry) > AV:B (Aviation with BRADSO). While this may be a transmutation of the "alternate system," I believe many cadets could utilize this system as it is the case that people view branch without ADSO and BRADSO slots are considered almost different things."

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- In the rest of my talk, I present the formal modeling and analysis that lead to the new branching mechanism the Army adopted both for USMA and ROTC starting with the Class of 2021.

Cadets, Branches & Terms

- I: Set of cadets
  - Each in need of at most one position at a branch
- $T = \{t^0, t^+\}$: Set of possible contractual terms to acquire a position
  - Elements are totally ordered
  - t^0: Base price
  - t^+: Increased price
- B: Set of branches
  - q_b: # of positions at branch $b \in B$
  - q_b^f: Maximum # that can be awarded at the increased price t^+

    Terminology: q_b^f positions are flexible-price

Cadet Preferences & Branch Baseline Priorities

- Cadet Preferences $\succ_i$: Linear order on $(B \times T) \cup \{\emptyset\}$
  - Assumption: For any cadet $i \in I$ and branch $b \in B$,

$$(b, t^0) \succ_i (b, t^+)$$

  - $\succeq_i$: Induced weak preference relation.
  - $\mathcal{Q}$: Set of resulting cadet preferences
- Branch Baseline Priorities π_b: Linear order on I
  - Π: Set of branch baseline priorities

Price Responsiveness Policy

- Price Responsiveness Policy $\omega_b^+(\pi_b)$: For a given $b \in B$ and $\pi_b \in \Pi$, a linear order on $I \times T$ with the following two properties:

  1. Same as the baseline priority order π_b for any fixed contractual term.

  For any $i, j \in I$ and $t \in T$,

$$(i,t)\,\omega_b^+\,(j,t) \quad \Longleftrightarrow \quad i\,\pi_b\,j$$

  2. Positively monotonic in contractual term for any given cadet.

  For any $i \in I$,

$$(i,t^+)\,\omega_b^+\,(i,t^0).$$

  - For the Army application, also called the BRADSO policy.
  - $\Omega_b^+(\pi_b)$: Set of resulting price responsiveness policies.
  - Identifies the priority upgrade gained for the flexible-price positions by paying the increased cost. Akin to Marginal Rate of Substitution.

Examples of Price Responsiveness Policies

- Ultimate Price Responsiveness Policy
  - The increased price grants any individual higher priority over any individual who pays the base price.
  - Used at USMA for Classes of 2006-2019.
- Tiered Price Responsiveness Policy
  - Individuals are partitioned into tiers within the baseline priority order.
  - Priority upgrade due to increased price is a function of tier.
  - Ultimate price responsiveness policy is a special case with a single tier.
  - Two distinct versions used at USMA for Classes of 2020 and 2021.
- Scoring-Based Price Responsiveness Policy
  - Baseline priority order is determined with a scoring rule.
  - Increased price grants a fixed boost to total score.
  - In the past used in various Chinese cities for their public high school admissions under the ZX Policy (Ze Xiao).

Outcome: A set of Contracts

- A contract is a triple $x \equiv \Big(\mathrm{i}(x), \mathrm{b}(x), \mathrm{t}(x)\Big) \in I \times B \times T$.
  - Interpretation: A position for cadet $\mathrm{i}(x)$ at branch $\mathrm{b}(x)$ at price $\mathrm{t}(x)$
  - $\mathcal{X} \equiv I \times B \times T$: Set of all contracts
  - $\mathcal{X}_i \equiv \{x \in \mathcal{X} : \mathrm{i}(x) = i\}$: Set of contracts that involve cadet i
  - $\mathcal{X}_b = \{x \in \mathcal{X} : \mathrm{b}(x) = b\}$: Set of contracts that involve branch b

- An allocation is a set of contracts $X \subset \mathcal{X}$, such that

  1. for any $i \in I$, $\Big|\{x \in X : \mathrm{i}(x) = i\}\Big| \leq 1,$
  2. for any $b \in B$, $\Big|\{x \in X : \mathrm{b}(x) = b\}\Big| \leq q_b,$ and
  3. for any $b \in B$, $\Big|\{x \in X : \mathrm{b}(x) = b \text{ and } \mathrm{t}(x) = t^+\}\Big| \leq q_b^f.$

  - $\mathcal{A}$: Set of allocations

Assignment

- For a given allocation $X \in \mathcal{A}$ and cadet $i \in I$, the assignment X_i of cadet i under allocation X is defined as

$$X_i = \begin{cases} (b,t) & \text{if } (i,b,t) \in X \\ \emptyset & \text{if } X \cap \mathcal{X}_i = \emptyset. \end{cases}$$

  - Slight abuse of notation: $\mathrm{b}(X_i)$ indicates the branch of assignment X_i
  - A cadet $i \in I$ is unmatched under allocation $X \in \mathcal{A}$ if $X_i = \emptyset$.

Mechanism

- A mechanism is a strategy space $\mathcal{S}_i$ for each cadet $i \in I$ along with an outcome function $\varphi : \prod_{i \in I} \mathcal{S}_i \to \mathcal{A}$ that selects an allocation for each strategy profile.
  - $\mathcal{S} \equiv \prod_{i \in I} \mathcal{S}_i$: Set of strategy profiles
- A mechanism $(\mathcal{S}, \varphi)$ is a direct mechanism, if $\mathcal{S}_i = \mathcal{Q}$ for each $i \in I$.
  - As it is customary, we denote a direct mechanism with its outcome function only.

Axioms

- Our approach is axiomatic. We formulate Army's policy objectives as technical axioms, and characterize the unique direct mechanism that satisfies all.
- All but one of our axioms are defined both for allocations and also for mechanisms.
  - Terminology: In those cases a mechanism satisfies the axiom if its outcome satisfies the axiom for all strategy profiles.

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- All but one of our axioms are defined both for allocations and also for mechanisms.
  - Terminology: In those cases a mechanism satisfies the axiom if its outcome satisfies the axiom for all strategy profiles.
- An allocation $X \in \mathcal{A}$ satisfies individual rationality if, for any $i \in I$,

$$X_i \succ_i \emptyset.$$

  - A mechanism $(\mathcal{S}, \varphi)$ satisfies individual rationality if the allocation $\varphi(s)$ satisfies individual rationality for any strategy profile $s \in \mathcal{S}$.

Axioms

- An allocation $X \in \mathcal{A}$ satisfies satisfies non-wastefulness if, for any $b \in B$ and $i \in I$,

$$\left.\begin{array}{c} \left|\{x \in X : \mathrm{b}(x) = b\}\right| < q_b\,, \quad \text{and} \\ X_i = \emptyset \end{array}\right\} \implies \emptyset \succ_i (b, t^0).$$

  - A mechanism $(\mathcal{S}, \varphi)$ satisfies non-wastefulness if the allocation $\varphi(s)$ satisfies non-wastefulness for any strategy profile $s \in \mathcal{S}$.

Axioms

- An allocation $X \in \mathcal{A}$ has no priority reversals if, for any $i, j \in I$, and $b \in B$

$$\left.\begin{array}{c} \mathrm{b}(X_j) = b \ \text{ and} \\ X_j \succ_i X_i \end{array}\right\} \implies j \, \pi_b \, i.$$

  - A mechanism $(\mathcal{S}, \varphi)$ has no priority reversals if the allocation $\varphi(s)$ has no priority reversals for any strategy profile $s \in \mathcal{S}$.

Axioms

- We next present two auxiliary definitions that highlight the intuition for our next axiom.
- Given an allocation $X \in \mathcal{A}$ and a cadet $i \in I$ with $\mathrm{t}(X_i) = t^+$, a cadet $j \in I \setminus \{i\}$ has a legitimate claim for a price-reduced version of cadet i's assignment X_i if,

$$\big(\mathrm{b}(X_i), t^0\big) \succ_j X_j \quad \textit{and}$$
$$(j, t^0)\ \omega^+_{\mathrm{b}(X_i)}\ (i, t^+).$$

  - Here cadet j's claim for a position at branch $\mathrm{b}(X_i)$ at the base price t^0 is legitimate, because the price responsiveness policy $\omega^+_{\mathrm{b}(X_i)}$ does not overturn her higher claim for a position at branch $\mathrm{b}(X_i)$ in favor of cadet i even when cadet i pays the increased price.

Axioms

- Given an allocation $X \in \mathcal{A}$ and a cadet $i \in I$ with $\mathrm{t}(X_i) = t^0$, a cadet $j \in I \setminus \{i\}$ has a legitimate claim for a price-increased version of cadet i's assignment X_i if,

$$\big(\mathrm{b}(X_i), t^+\big) \succ_j X_j \,,$$
$$(j, t^+)\; \omega^+_{\mathrm{b}(X_i)}\; (i, t^0)\,, \quad \textit{and}$$
$$\Big(X \setminus \big\{\big(i, \mathrm{b}(X_i), t^0\big)\big\}\Big) \bigcup \big\{\big(j, \mathrm{b}(X_i), t^+\big)\big\} \in \mathcal{A}.$$

  - Here cadet j's claim for a position at branch $\mathrm{b}(X_i)$ at the increased price t^+ is legitimate, because, even if cadet i has a higher baseline priority at branch $\mathrm{b}(X_i)$,
    1. the price responsiveness policy $\omega^+_{\mathrm{b}(X_i)}$ overturns this priority in favor of cadet j for as long as cadet j pays a higher price than cadet i, and
    2. awarding the position originally given to cadet i instead to cadet j albeit at a higher price t^+ is feasible and it does not result in exceeding the cap $q^f_{\mathrm{b}(X_i)}$ for flexible-price positions at branch $\mathrm{b}(X_i)$.

Axioms

- We are ready to present the key axiom which differentiates our analytical results from earlier results in the literature.
- An allocation $X \in \mathcal{A}$ satisfies enforcement of the price responsiveness policy if, no cadet $j \in I$ has a legitimate claim for either a price-reduced version or a price-increased version of the assignment X_i of another cadet $i \in I \setminus \{j\}$.
  - A mechanism $(\mathcal{S}, \varphi)$ satisfies the enforcement of the price responsiveness policy if the allocation $\varphi(s)$ satisfies the enforcement of the price responsiveness policy for any strategy profile $s \in \mathcal{S}$.

Axioms

- Our last axiom is a highly sought-after incentive-compatibility property, defined for direct mechanisms only.

- A direct mechanism φ is strategy-proof if, for any $\succ \in \mathcal{Q}^{|I|}$, any $i \in I$, and any $\succ_i' \in \mathcal{Q}$,
$$\left[\varphi(\succ)\right]_i \succeq_i \left[\varphi(\succ_{-i}, \succ_i')\right]_i$$

Dual-Price Cumulative Offer Mechanism

- The Dual-Price Cumulative Offer (DPCO) mechanism is a direct mechanism based on the seminal cumulative offer procedure (Hatfield & Milgrom 2005) together with the following choice rule for each branch.
- Dual-Price Choice Rule $\mathcal{C}_b^{DP}$: Given a branch $b \in B$ and set of contracts $X \in \mathcal{X}_b$, select (up to) q_b contracts with distinct cadets in two steps as follows:

  Step 1. For the base-price positions, exclusively select base-price contracts with the highest baseline priority cadets.

  Step 2. For the flexible-price positions , select the highest-priority remaining contracts based on the price responsiveness policy ω_b^+.

Dual-Price Cumulative Offer Mechanism

- Fix any linear order of cadets, say the OML. (This linear order does not affect the outcome by Kominers & Sönmez 2016).
- At any step ℓ of the procedure,
  - the highest-OML cadet i_ℓ who currently has no contract on hold offers his most-preferred previously-unrejected contract x_ℓ to the branch of the contract $\mathrm{b}(x_\ell)$, and
  - considering all offers X_ℓ it has received up to (and including) Step ℓ, branch $\mathrm{b}(x_\ell)$ holds the contracts in $\mathcal{C}^{DP}_{\mathrm{b}(x_\ell)}(X_\ell)$, and rejects all others.
- The procedure terminates when either no cadet remains with an acceptable contract that has not been rejected, or when no contract is rejected. All the contracts on hold in the final step are finalized.

Main Characterization Result

Theorem

Fix a profile of baseline priority orders $(\pi_b)_{b\in B} \in \Pi$ and a profile of price responsiveness policies $\left(\omega_b^+\right)_{b\in B} \in \prod_{b\in B} \Omega_b^+$. A direct mechanism φ satisfies

1. *individual rationality,*
2. *non-wastefulness,*
3. *enforcement of the price responsiveness policy,*
4. *strategy-proofness, and*
5. *has no priority reversals,*

if and only if

$$\varphi = DPCO$$

Significance of the Characterization

- Prior to our analysis, Hirata & Kasuya (2017) and Hatfield, Kominers & Westcamp (forthcoming) presented earlier characterizations of the cumulative offer mechanism.
  - Fundamentally different than our analysis, each institution is endowed with an exogenously given choice rule that satisfies various technical conditions in these papers.
  - In our characterization, in contrast, the dual-price choice rule emerges endogenous to the Army's policy objectives.
  - Hence, from a theoretical perspective, our main theorem is the first joint characterization of the cumulative offer mechanism together with a specific choice rule.
  - And, from a practical perspective, it has been instrumental in the joint design of the Army's new allocation mechanism and its priority system.

Significance of the Characterization

- Indeed, the very concept of a choice rule is merely used in our model to describe the DPCO mechanism.
    - Not only our axioms do not rely on any structure or functional form of potential branch choice rules, even the existence of a well-defined choice rule for any given branch is not assumed in our analysis.
    - Instead, the dual-price choice rule emerges from our analysis in tandem with the cumulative offer mechanism as a collective implication of our five axioms.
    - This is why our result is a characterization of a refinement of the cumulative offer mechanism rather than another characterization of the cumulative offer mechanism itself.

Significance of the Characterization

- It is also important to emphasize that all our axioms reflect the Army's policy objectives, and none of them are are imposed upon as technical conditions for the sake of obtaining an axiomatic characterization.
- On the contrary, the very reason the Army has initiated a collaboration with the two civilian co-authors of this paper is the design of a branching system which ideally satisfies all these axioms.
- As I have have emphasized earlier in the presentation, starting with 2006 the Army's adjustments in its branching mechanisms to implement its BRADSO policies have resulted in priority reversals along with incentive compatibility failures.
- In a manner of speaking, the effort to accommodate the enforcement of the price responsiveness policy axiom has resulted in an unintended consequence of the failure of two other key axioms.

Other Applications

- Wang and Zhou (2020): Public high school admissions in China under the ZX Policy (Ze Xiao).
  - A fraction of the seats are available with an increased tuition.
  - Baseline priorities are based on scores on a centralized exam.
  - The higher-tuition contract increases this score by a fixed amount for the ZX-eligible seats.
  - Shanghai and Tianjin both have a single ZX tuition level, making these applications completely analogous to the Army's problem.
  - In some cities there were multiple tuition levels where higher tuition levels result in higher adjustments to student score.

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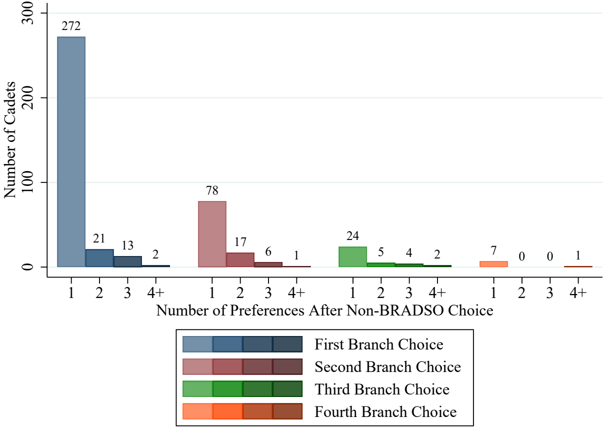
  - In some cities there were multiple tuition levels where higher tuition levels result in higher adjustments to student score.
  - Discontinued after 2015.

Other Applications

- Aygün and Turhan (2020): Affirmative Action (AA) with Stigma.
  - A fraction of the seats at each school give preferential treatment to AA-eligible students.
  - For any given school, any AA-eligible student prefers receiving a seat without invoking the AA privileges to receiving a seat via AA.
  - Analogous to the Army's problem under the ultimate BRADSO policy.

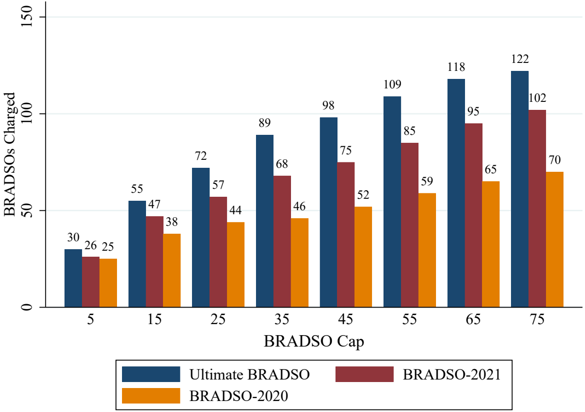
Cadet Utilization of the New Strategy Space

- Our main theorem highlights that, the key decision by the Army was the adoption of the new strategy space. The following figure reports the extent of its utilization by the Class of 2021.



Army's Selection of BRADSO Policy and Cap

- In order to determine the number of flexible-price positions and the BRADSO-policy, Army relied on simulations with data from earlier years.
- The following figure reports simulations of the DPCO mechanism with data from the actual run of the mechanism for Class of 2021.



Conclusion

- Army considered the design a success, and also adopted it for ROTC ahead of its scheduled time.
    - The decision to use DPCO mechanism for ROTC was in part due to concerns that ROTC's previous branching mechanism generated dead zones that made priority reversals particularly visible, as discussed in Sönmez (2013).
- Army has also identified additional ways to utilize the cumulative offer mechanism.
- Our model and the DPCO mechanism can be used in other applications where an agent can take a costly action in exchange for higher priority in a fraction of positions.

Conclusion

- On a broader level, our paper highlights
  - the importance of theory in market design, and
  - illustrates the practical value of the matching with contracts model.

  Without the following fundamental contributions in pure theory, Army's reform would not happen:

  - Gale & Shapley (1962)
  - Kelso & Crawford (1982)
  - Hatfield & Milgrom (2005)
  - Hatfield & Kojima (2010)
  - Echenique (2012)