AFFIRMATIVE ACTION IN INDIA VIA VERTICAL AND HORIZONTAL RESERVATIONS

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ABSTRACT. Built into the country's constitution, one of the world's most comprehensive affirmative action programs exists in India. Government jobs and seats at publicly funded educational institutions are allocated through a Supreme Court-mandated procedure that integrates a meritocracy-based system with a reservation system that provides a level playing field for disadvantaged groups through two types of special provisions. The higher-level provisions, known as vertical reservations, are exclusively intended for backward classes that faced historical discrimination, and implemented on a "set aside" basis. The lower-level provisions, known as horizontal reservations, are intended for other disadvantaged groups (such as women or disabled citizens), and they are implemented on a "minimum guarantee" basis. We show that, the Supreme Court-mandated procedure suffers from two major deficiencies: Not only a candidate can lose a position to a less meritorious candidate from a higher-privilege group, completely against the philosophy of affirmative action, but she can also lose a position simply because of disclosing her disadvantaged group. This loophole under the Supreme Court-mandated procedure causes widespread confusion in India, resulting in countless lawsuits, conflicting judgements on these lawsuits, and even defiance in some of its states. A recent amendment in the Constitution of India has the potential to amplify the adverse effects of these shortcomings. We propose an alternative procedure that resolves both deficiencies with the smallest possible deviation from the Supreme Court-mandated procedure.

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1. Introduction

While the term "affirmative action" was first used in 1961 when President John F. Kennedy signed Executive Order 10925, the 1950 Constitution of India had already mandated affirmative action to the members of its so-called "backward classes." The intended groups were Scheduled Castes (SC), which is the official term for Dalits or "untouchables," whose members have suffered millenniums-long systematic injustice due to their lowest status under the caste system, and Scheduled Tribes (ST), which is the official term for the indigenous ethnic groups of India, whose members were both physically and socially isolated from the rest of the society. Built into the country's constitution, affirmative action has been implemented in India through a reservation system that earmarks a certain percentage of government jobs and university seats, initially only to the members of SC and ST, and eventually to the members of Other Backward Classes (OBC) as well. Indeed, Article 16(4) of the Constitution of India reads:

> Nothing in this article shall prevent the State from making any provision for the reservation of appointments or posts in favor of any backward class of citizens which, in the opinion of the State, is not adequately represented in the services under the State.

In addition to this article, under which the intended beneficiaries are exclusively the members of backward classes, certain provisions are allowed under Article 16(1) for other groups of disadvantaged individuals—such as disabled citizens—to promote equality of opportunity.

While embedded in its 1950 Constitution, the scale, scope, and mechanisms of affirmative action in India have always been highly contested. As a result, the judiciary has always taken an active role in its implementation and enforcement. The following statement from the Supreme Court judgement *Indra Sawhney and others v. Union of India (1992)*,[1] one of the most influential judgements in the history of India, summarizes the sentiment on this important topic:

> The questions arising herein are not only of great moment and consequence, they are also extremely delicate and sensitive. They represent complex problems of Indian Society, wrapped and presented to us as constitutional and legal questions...
>
> There are occasions when the obvious needs to be stated and, we think, this is one such occasion. We are dealing with complex social, constitutional and legal questions upon which there has been a sharp division of opinion in the Society, which could have been settled more

[1]The case is available at `https://indiankanoon.org/doc/1363234/` (last accessed on 03/10/2019).

> satisfactorily through political processes. But that was not to be. The issues have been relegated to the judiciary...
>
> There are other reasons, of course - that cause governments to leave decisions to be made by Courts. They are of expedient political character. The community may be so divided on a particular issue that a government feels that the safe course for it to pursue is to leave the issue to be resolved by the Courts, thereby diminishing the risk it will alienate significant sections of the Community.

India is a federal union that consists of twenty-nine states with a unitary, three-tiered judiciary made up of lower trial courts, a high court for each state, and a Supreme Court above all courts. The Supreme Court is not only vested with original jurisdiction to issue writs in defence of the fundamental rights listed in the Constitution, but also with appellate jurisdiction from the high courts to review and change the outcomes of their decisions (Neuborne, 2003). As a result, the Supreme Court of India has always played a central role in matters of affirmative action.

In *Indra Sawhney (1992)*, the Constitution bench of the Supreme Court formulated *vertical reservations* (also called *social reservations*) as a tool to implement the higher-level provisions enabled by Article 16(4), and *horizontal reservations* (also called *special reservations*) as a tool to implement the lower-level provisions enabled by Article 16(1). The scope and the mechanics of these two types of reservations were distinctly differentiated in this judgement as follows:

(1) *Vertical reservations*:
   (a) They are the highest form of special provisions that are intended exclusively for members of backward classes SC, ST, and OBC.
   (b) Being the highest form of special provisions, these reserved positions are to be earmarked to the members of backward classes in the form of a "set aside," which means positions secured by members of these classes on the basis of their own merit are not counted against vertically reserved positions.
   (c) They cannot exceed 50% of the positions.[2]

[2]Not all states follow the 50% upper bound for vertical reservations. Most notable example is Tamil Nadu with 69.5%. See *The Print* story "4 states have gone over SC-imposed 50 percent reservation cap. Will Rajasthan follow?" available at `https://theprint.in/india/governance/will-rajasthan-exceed-sc-imposed-50-per-cent-reservation-cap/16965/` (last accessed on 3/27/2019). Moreover the *One Hundred and Third Amendment of the Constitution of India* that came into effect in India on 01/14/2019 provides 10% reservation to the economically weaker sections (EWS), increasing the percentage to vertical reservations to 60%. See Section 7 for the impact of this recent amendment on our analysis.

(2) *Horizontal reservations*:[3]
  (a) They are the lesser form of special provisions that are intended for other disadvantaged groups of citizens (disabled, women, etc.), and adjustments through them cannot interfere with the number of positions vertically reserved for the backward classes.
  (b) They are provided as a "minimum guarantee," which means positions secured by horizontal reserve-eligible candidates on the basis of their own merit, and, thus, without using the benefit of horizontal reservations, nonetheless are counted against horizontally reserved positions.

In the absence of horizontal reservations, implementation of vertical reservations is a straightforward task. First, open positions are to be filled one at a time according to merit score (including those candidates from SC, ST, and OBC), and next for each of the backward classes SC, ST, and OBC, the vertically reserved positions are to be filled one at a time with the remaining candidates of the given backward class, based on merit score.

In many applications, however, there are also horizontal reservations,[4] and in this case how exactly to integrate these two types of provisions is less clear. While the principles that dictate the implementation of reservations were clearly laid out in *Indra Sawhney (1992)*, an explicit procedure to implement them was not provided. Perhaps due to a large number of lawsuits brought to high courts and the Supreme Court, an explicit procedure was provided to this end in another judgement of the Supreme Court, in *Anil Kumar Gupta v. State of U.P. (1995)*.[5] This judgement has been used as a main reference in virtually all subsequent legal disputes—thousands of them—on integrated implementation of vertical and horizontal reservations. Indeed, it is referred to as a "class by itself" by the judges of the Madras High Court in their case *K.R.Shanthi vs The Secretary To Government (2012)*:[6]

> `This judgment is a class by itself which clearly makes a demonstration as to how selection has to be made as against the open quota and the reserved quota for various reserved classes by applying the vertical reservation and special reservation for women, physically handicapped etc., by following horizontal reservation.`

[3]It is important to emphasize that, even though the special provisions covered by Article 16(1) are referred to as horizontal reservations, they are not considered reservations. In its judgement *Indra Sawhney (1992)* the judges of the Supreme Court clarifies this technical distinction as follows: "Article 16(4) being part of the scheme of equality doctrine it is exhaustive of reservation, therefore, no reservation can be made under Article 16(1)." Hence the phrase reservation, when used alone, always refers to vertical reservation.

[4]For example, having a disability is federally mandated as a trait by the Supreme Court in their judgement of *Union Of India & Anr vs National Federation Of The Blind & ... on 8 October, 2013*. The case is available at `https://indiankanoon.org/doc/178530295/` (last accessed on 03/14/2019).

[5]The case is available at `https://indiankanoon.org/doc/1055016/` (last accessed on 03/10/2019).

[6]The case is available at `https://indiankanoon.org/doc/41866200/` (last accessed on 03/10/2019).

To the best of our knowledge, our paper is the first one that formulates and analyses vertical and horizontal reservations, when both types of provisions coexist. From a market-design angle, we show that the reference procedure given in *Anil Kumar Gupta (1995)* and mandated throughout India has two important shortcomings. Our main contributions are:

(1) formulating these shortcomings in Section 4.1,
(2) documenting that they are responsible for numerous lawsuits throughout India in Section 6, and
(3) resolving them through an alternative procedure in Section 5.

We also relate these shortcomings to the *One Hundred and Third Amendment of the Constitution of India* in Section 7, and argue that the adverse impact of these shortcomings will likely increase considerably since the amendment interacts rather poorly with the vulnerabilities of the Supreme Court-mandated procedure.

While vertical and horizontal reservations are introduced to protect disadvantaged groups, the Supreme Court-mandated procedure allows for situations where a candidate from a disadvantaged group, despite being more meritorious, may still lose a position to a candidate from a more privileged group. We refer to this anamoly as a failure to *eliminate justified envy*. This failure is highly inconsistent with the principle of *inter se* merit built into the Constitution of India, whereby a candidate can never lose a position to a less meritorious candidate provided that they are from the same group. Indeed, under the Supreme Court-mandated procedure a candidate can never lose a position to a less meritorious candidate from the same group, but ironically she can lose a position to a less meritorious candidate from a more privileged group. In addition to this highly implausible possibility, the Supreme Court-mandated procedure may also penalize candidates for reporting their vertical reserve-eligible backward class. In that sense, the procedure is not *incentive compatible*. These two shortcomings not only result in countless lawsuits, but also provide a loophole in the procedure that can be used to discriminate against members of backward classes. In Section 6, we provide ample evidence that these shortcomings are responsible for widespread confusion in India, often resulting in legal action, and even defiance in some states through the illegal implementation of better-behaved versions of the mandated procedure. We also provide evidence in Section 6.2 that, in some jurisdictions these shortcomings are exploited by local officials to discriminate against members of backward classes. These litigations often result in the interruption of the recruitment process, as well as reversals of recruitment decisions. Reporting a judgement by the Gujarat High Court, an article in *The Times of India* highlights this very issue:[7]

[7] *The Times of India* story is available at `https://timesofindia.indiatimes.com/city/ahmedabad/general-seat-vacated-by-quota-candidate-remains-general-hc/articleshowprint/57658109.cms` (last accessed on 04/12/2019).

> The advertisement was issued in 2010 and recruitment took place in 2016 amid too many litigations over the issue of reservation ...
>
> With the recent observation by the HC, the merit list will now be changed for the third time. Those already selected and at present under training might lose their jobs, and half a dozen new candidates might find their names on the new list. However, all appointments have been made by the HC conditionally and subject to final outcome of these multiple litigations.

Motivated by the shortcomings of the Supreme Court-mandated procedure, we propose in Section 5 an alternative choice rule. This alternative choice rule not only eliminates these shortcomings (Proposition 4), but it is also a "minimal deviation" from the Supreme Court-mandated procedure in the sense that, outcomes of the two procedure only differ when the outcome of the Supreme Court-mandated procedure fails to eliminate justified envy (Theorem 1).

A simple search of the phrase "horizontal reservation" via Indian Kanoon, a free search engine for Indian Law, reveals the scale of the litigations relating this concept. As of 10/10/2019, there are 1656 cases at the Supreme Court and State High Courts where the dispute relate to the concept of horizontal reservation.[8] These cases do not include those at the lower courts.

1.1. Related Literature. While there is a rich literature on affirmative action policies in India and elsewhere, to the best of our knowledge, our paper is the first one to formally analyze vertical and horizontal reservation policies when they are jointly implemented.

There are a number of recent papers on reservation policies, most in the context of school choice. Abdulkadiroğlu and Sönmez (2003) study affirmative action policies that limit the number of students of a given type at schools. Kojima (2012) shows that the policy of limiting the number of majority students can hurt minority students, the intended beneficiaries. To overcome the detrimental effect of affirmative action policies based on majority quotas, Hafalir et al. (2013) introduce affirmative action policies based on minority reserves. Echenique and Yenmez (2015) study when there can be reservations for every type of student and provide an axiomatic characterization of choice rules with horizontal reservations that provide a minimum guarantee for students. We show in Proposition 2 that when each candidate qualifies for at most one trait of horizontal reservation, the unique merit-maximal choice rule that we construct is the same as the choice rule of

[8] Not all cases on "horizontal reservation" is about disputes related to elimination of justified envy or incentive compatibility. However during our search, we observed that the terminology of "migration" was used is some of the cases to indicate the situations where members of reserved categories were allowed for horizontal adjustments of open-category positions, and the more refined search of "horizontal reservation, migration" generated 245 cases at the Supreme Court and High Courts. As far as we can tell, a vast majority of these cases relate to the shortcomings on elimination of justified envy or incentive compatibility.

Echenique and Yenmez. Ehlers et al. (2014) study more general affirmative action policies that adjust the priorities of students depending on the number of admitted students with different types.

Reservation policies studied in two recent papers can be interpreted as special cases of policies studied in our paper. In Dur et al. (2016), the authors study allocation of Chicago's elite public high school seats to eighth graders, and compare various reservation policies. In Chicago, students are partitioned into four socio-economic classes based on their home addresses, and 17.5% of the seats are reserved for each socio-economic class as a set aside (i.e., as a vertical reservation), while the remaining 30% is open to all students and priority is based on their composite scores. The authors show that under some distributional assumptions on composite scores, and fixing the numbers of reserve seats, Chicago's policy is the best possible policy for the members of the lowest socio-economic class. In Dur et al. (2018), the authors study allocation of Boston's public school seats to students, where 50% of the seats at each school are reserved for neighborhood students as a minimum guarantee (i.e., on a horizontal reservation basis). The authors show that implementation of walk-zone seats on a minimum guarantee basis was inconsistent with the stated allocation policy in Boston, and it had the unintended consequence of virtually eliminating its walk-zone priorities. Based on this result, the city has given up walk-zone priorities altogether starting with 2013-14 school year, in an effort to increase the system's transparency. Both of these models are applications of the more general model in Kominers and Sönmez (2016), where the authors introduce a matching model with slot-specific priorities. Unless there is at most one trait of horizontal reservation, our model cannot be covered by Kominers and Sönmez (2016).

Three additional papers on reservation policies include Aygün and Turhan (2016, 2017), where the authors study admissions to engineering colleges in India, and Aygün and Bó (2016), where the authors study admissions to Brazilian public universities. While the application in Aygün and Turhan (2016, 2017) is closely related to ours, their analysis is independent because they assume away horizontal reservations altogether. Indeed, the shortcomings we formulate in our paper disappear in the absence of horizontal reservations. The Brazilian affirmative action application studied by Aygün and Bó (2016) relates to ours in that it also includes multi-dimensional reserves, but unlike ours their application is a special case of Kominers and Sönmez (2016).[9]

More broadly, our paper contributes to market design, where economists are increasingly taking advantage of advances in technology to design new or improved allocation mechanisms in applications as diverse as entry-level labor markets (Roth and Peranson, 1999), school choice (Balinski and Sönmez, 1999; Abdulkadiroğlu and Sönmez, 2003),

[9] Other related papers on design of reservation policies include Westkamp (2013), Kamada and Kojima (2015), and Fragiadakis and Troyan (2017).

spectrum auctions (Milgrom, 2000), kidney exchange (Roth et al., 2004, 2005), internet auctions (Edelman et al., 2007; Varian, 2007), course allocation (Sönmez and Ünver, 2010; Budish, 2011), cadet-branch matching (Sönmez and Switzer, 2013; Sönmez, 2013), assignment of arrival slots (Schummer and Vohra, 2013; Schummer and Abizada, 2017), refugee matching (Jones and Teytelboym, 2017; Delacrétaz et al., 2016; Andersson, 2017), and interdistrict school choice (Hafalir et al., 2018).

2. Institutional Background on Vertical and Horizontal Reservations

In *Indra Sawhney (1992)*, the Constitution bench of the Supreme Court coined the terms *vertical reservation* and *horizontal reservation*, while emphasizing in the following statement how these two types of affirmative action tools are to interact with each other:

> `A little clarification is in order at this juncture: all reservations are not of the same nature. There are two types of reservations, which may, for the sake of convenience, be referred to as 'vertical reservations' and 'horizontal reservations'. The reservation in favour of scheduled castes, scheduled tribes and other backward classes [under Article 16(4)] may be called vertical reservations whereas reservations in favour of physically handicapped [under clause (1) of Article 16] can be referred to as horizontal reservations. Horizontal reservations cut across the vertical reservations -- what is called interlocking reservations. To be more precise, suppose 3% of the vacancies are reserved in favour of physically handicapped persons; this would be a reservation relatable to clause (1) of Article 16. The persons selected against his quota will be placed in the appropriate category; if he belongs to SC category he will be placed in that quota by making necessary adjustments; similarly, if he belongs to open competition (OC) category, he will be placed in that category by making necessary adjustments.`

It is further emphasized in the judgement that vertical reservations in favor of backward classes SC, ST, and OBC (which the judges refer to as *reservations proper*) are earmarked for these classes, and they cannot be reduced due to positions allocated through horizontal reservations.

While horizontal reservations can be implemented either as *overall horizontal reservations* for the entire set of positions, or as *compartment-wise horizontal reservations* within each vertical category including the open category (OC), the Supreme Court recommended the latter in their judgement of *Anil Kumar Gupta (1995)*:

> `We are of the opinion that in the interest of avoiding any complications and intractable problems, it would be better that in future the horizontal reservations are comparmentalised in the sense explained above. In other`

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words, the notification inviting applications should itself state not
only the percentage of horizontal reservation(s) but should also specify
the number of seats reserved for them in each of the social reservation
categories, viz., S.T., S.C., O.B.C. and O.C.
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The compartment-wise implementation of horizontal reservations ensures that, unlike the aforementioned case, the distributional benefits of the special horizontal reservations extend to all segments of the society. Consistent with the Supreme Court's recommendation, many states in India have adopted compartment-wise implementation of horizontal reservations in their allocation of public positions. For example, in an effort to increase the participation of women in public employment, compartment-wise horizontal reservations for female candidates is mandated by government order in several states, including in Bihar with 35%, Andhra Pradesh with $33\frac{1}{3}$%, and Madhya Pradesh, Uttarakhand, Chhattisgarh, Rajasthan, and Sikkim with 30% each. As such, we will focus on compartment-wise horizontal reservations. That is, the term horizontal reservation will indicate its compartment-wise implementation throughout the paper.

2.1. Implementation of Vertical and Horizontal Reservation: SCI-VHR Choice Rule. The judges of the Supreme Court did not merely specify the principles that govern the implementation of the social vertical and special horizontal reserves; they also provided a procedure to implement these reserves in their judgement of *Anil Kumar Gupta (1995)*. The procedure provided by the Supreme Court in this case, using the court's own wording, is as follows:

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The proper and correct course is to first fill up the O.C. quota (50%)
on the basis of merit: then fill up each of the social reservation
quotas, i.e., S.C., S.T. and B.C; the third step would be to find out
how many candidates belonging to special reservations have been selected
on the above basis. If the quota fixed for horizontal reservations is
already satisfied - in case it is an over-all horizontal reservation
- no further question arises. But if it is not so satisfied, the
requisite number of special reservation candidates shall have to be taken
and adjusted/accommodated against their respective social reservation
categories by deleting the corresponding number of candidates therefrom.
(If, however, it is a case of compartmentalised horizontal reservation,
then the process of verification and adjustment/accommodation as stated
above should be applied separately to each of the vertical reservations.
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The adjustment phase of the procedure for special horizontal reserves is further elaborated in the Supreme Court judgement *Rajesh Kumar Daria v. Rajasthan Public Service Commission and others (2007)* as follows:[10]

> If 19 posts are reserved for SCs (of which the quota for women is four), 19 SC candidates shall have to be first listed in accordance with merit, from out of the successful eligible candidates. If such list of 19 candidates contains four SC women candidates, then there is no need to disturb the list by including any further SC women candidate. On the other hand, if the list of 19 SC candidates contains only two woman candidates, then the next two SC woman candidates in accordance with merit, will have to be included in the list and corresponding number of candidates from the bottom of such list shall have to be deleted, so as to ensure that the final 19 selected SC candidates contain four women SC candidates. [But if the list of 19 SC candidates contains more than four women candidates, selected on own merit, all of them will continue in the list and there is no question of deleting the excess women candidate on the ground that 'SC-women' have been selected in excess of the prescribed internal quota of four.]

We refer to this choice rule as the *Supreme Court of India Vertical & Horizontal Reservations choice rule*, or *SCI-VHR choice rule* in short.

We are ready to present our formal model, theoretical results, and policy recommendations.

3. Model and Preliminary Results

Consider a finite set of individuals $\mathcal{I}$ who apply for q identical positions. Each individual either belongs to a reserve-eligible category such as "Scheduled Castes" (SC), "Scheduled Tribes" (ST), and "Other Backward Classes" (OBC), or belongs to the "General" category (G). For each reserve-eligible category, a number of positions is earmarked exclusively for the members of this category. In contrast, there are no positions earmarked for the members of the general category. Denote the set of reserve-eligible categories by $\mathcal{C}$.

Category membership is denoted by a function ρ. For an individual $i \in \mathcal{I}$ and a reserve-eligible category $c \in \mathcal{C}$, let $\rho(i) = \{c\}$ indicate that i is a member of the category c. For an individual i, let $\rho(i) = \emptyset$ indicate that i is a member of the general category. For every set of individuals $I \subseteq \mathcal{I}$, let I^G denote the subset of general-category individuals in I and $I^R \equiv I \setminus I^G$ denote the subset of individuals in I with a reserve-eligible category.

In addition to being a member of a category, each individual also has a (possibly empty) set of traits. Each trait represents a disadvantage in the society, and the government may

[10] The case is available at `https://indiankanoon.org/doc/698833/` (last accessed on 03/12/2019).

provide individuals who have this trait with easier access to positions to level the playing field. The set of traits is finite and denoted by $\mathcal{T}$. The set of traits of individual i is denoted by $\tau(i) \subseteq \mathcal{T}$.

Finally, each individual has a distinct merit score, where the score of individual i is denoted as $\sigma(i) \in \mathbb{R}_+$.

An **allocation problem** is given by a tuple $\langle \mathcal{I}, \mathcal{C}, \mathcal{T}, \rho, \tau, \sigma, q \rangle$.

Given an allocation problem, a **choice rule** is a function C such that for any $I \subseteq \mathcal{I}$

$$C(I) \subseteq I \text{ with } |C(I)| \leq q.$$

In words, for a given number of positions, q, and a set of individuals I who are applying for the positions, the choice rule C produces a subset of individuals who are allocated these positions. In addition, the set of individuals in I who are not chosen by C is denoted by $R(I)$. The choice rule may also depend on the affirmative action policies, which we explain next.

3.1. Vertical Reservations Only. Affirmative action for the social categories is implemented by setting aside a number of positions for each category. These reservations are called *vertical* or *social*. For any category $c \in \mathcal{C}$, let r^c denote the number of positions set aside for individuals from category c. The remaining $r^o = q - \sum_c r^c$ positions are open for all individuals, and they are called open-category positions. When there are only vertical reservations, open-category positions are allocated based exclusively on merit scores.

When an individual from a reserve-eligible category receives an open-category position on his own merit, that does not count against the vertical reservations for his social category. This is the sense in which vertically reserved positions are "set aside" for members of reserve-eligible categories, regardless of who receives open-category positions.

In the absence of horizontal reservations introduced in Section 3.2, the following two federally-mandated principles define a unique subset of individuals that must be chosen from any set of applicants, provided that all positions are filled. First, an allocation must respect *inter se* merit: if a category-c individual with a lower merit score is given a position, then a category-c individual with a higher merit score must also be given a position. Furthermore, when an individual with a reserve-eligible category claims an open-category position by merit, he does not count for the vertical reservations for his category. Therefore, individuals with the highest merit scores must be allocated the open-category positions first, to determine which individuals are eligible for these positions. Then, positions reserved for the social categories must be allocated to the remaining individuals, again based on their merit scores.

3.2. Horizontal Reservations Only. With the Article 16(1) of the 1950 Constitution of India, disadvantaged individuals with certain traits are provided with some lower-level provisions referred to as *horizontal* or *special reservations*. These reservations provide a minimum guarantee for the number of individuals with these traits who are allocated positions. While horizontal reservations are provided in India at each vertical category, in this subsection we consider the basic case when all positions are open. Let $r_t > 0$ be the number of reserved positions for trait t, where $\sum_{t \in \mathcal{T}} r_t \leq q$.

Let $I \subseteq \mathcal{I}$ be a set of individuals who apply for a position. Say that $I' \subseteq I$ **satisfies trait-t reservations for I**, if, either

(1) the number of trait-t individuals in I' is at least r_t, or
(2) all trait-t individuals in I are included in I'.

Say that $I' \subseteq I$ **satisfies the horizontal reservations for I** if, for every trait t, I' satisfies trait-t reservations for I.

Consider the following choice rule.

Choice Rule $\mathbf{C^{hor}}$

Step 1: Consider all subsets of I that satisfy the horizontal reservations for I. Choose the individual with the highest merit score who is in any of these subsets. Let I_1 denote the set including only this individual.

Step k ($\mathbf{k} \in [\mathbf{2}, \mathbf{q}]$): Consider all subsets of I that include I_{k-1} and satisfy the horizontal reservations for I. If the only such subset is I_{k-1}, then stop and return this set. Otherwise, from $I \setminus I_{k-1}$, choose the individual with the highest merit score who is in any of these subsets. Let I_k denote the set of individuals chosen so far.

When the number of applicants is less than q, then this procedure chooses all the applicants. If there are more than q applicants, then it stops at Step q, and returns I_q which has q applicants. The outcome of this choice rule is denoted by $C^{hor}(I|q, (r_t)_{t \in \mathcal{T}})$, or simply by $C^{hor}(I)$ when there is no ambiguity about the reservation parameters.

Consider two different sets of individuals I and I'. Say that I **dominates** I' if, there exists an individual in $I \setminus I'$ with a merit score that is strictly greater than the merit scores of all individuals in $I' \setminus I$. Domination is a binary relation that one can use to compare different sets based on merit scores of individuals. It is easy to see that domination is a *strict partial order*.[11]

Given a set of applicants, there are typically multiple subsets of these applicants that satisfy the horizontal reservations. One possible way to determine which subset of applicants are "more deserving" of the positions is, using the domination relation.

Definition 1. *A choice rule C is **merit maximal** if, for every set of individuals I,*

(1) $C(I)$ satisfies the horizontal reservations for I, and
(2) $C(I)$ dominates I' for any other set $I' \subseteq I$ that satisfies the horizontal reservations for I.

[11]A binary relation is a strict partial order if it is *irreflexive*, *transitive*, and *asymmetric*.

We do not take any position on whether merit maximality is the most adequate way to determine the set of "most deserving" candidates. In Section 4, however, we present how this notion is utilized under the Supreme Court-mandated choice rule.

We are ready to present our first result.

Proposition 1. *C^{hor} is the unique merit-maximal choice rule.*

Note that this result holds regardless of the set of traits each individual has. However, when each individual has at most one trait, a widespread practice in India,[12] there is a simpler choice rule that gives the unique merit-maximal outcome. We refer to this alternative choice rule as C^{mg}, since it first accommodates the "minimum guarantees" for each trait, and then fills all the remaining positions.

Proposition 2. *Suppose that each individual has at most one trait. Then C^{hor} is equivalent to the following choice rule.*

Choice Rule $\mathbf{C^{mg}}$

Step 1: *For each trait $t \in \mathcal{T}$, if the number of trait-t individuals is less than r_t, then choose all of them. Otherwise, if this number is at least r_t, then choose trait-t individuals with the r_t highest merit scores.*

Step 2: *For the unfilled positions, choose the remaining individuals with the highest merit scores.*

Note that C^{mg} is not well-defined when individuals have multiple traits, because the order in which traits are processed in Step 1 is not specified. However, the order becomes immaterial when each individual has at most one trait, and in this case, C^{mg} is equivalent to the unique merit-maximal choice rule C^{hor}.[13] In Appendix D, we present a High Court case from Chhattisgarh, where the simplification established by Proposition 2 could have been utilized for a straightforward resolution.

3.3. Vertical and Horizontal Reservations. We are ready to introduce the model in its full generality, with both vertical and horizontal reservations. For a social category $c \in \mathcal{C}$, we refer to the positions vertically reserved for its members as category-c positions. Similarly, positions open for individuals from all categories are referred to as open-category

[12] As of 10/10/2019, the only federally mandated horizontal trait is for disabled individuals.

[13] This choice rule was first introduced in Echenique and Yenmez (2015) and is conceptually related to the slot-specific choice rule defined by Kominers and Sönmez (2016). The relation can be seen as follows. For each trait t, let $q_t \equiv \min\{r_t, \{i \in I | t \in \tau(i)\}\}$ be the number of slots reserved for trait-t individuals and $q^o \equiv q - \sum_t q_t$ be the number of open slots. For trait-t slots, only trait-t individuals are ranked, while for open slots all individuals are ranked according to their merit scores. First, slots reserved for traits are filled, after which open slots are filled. In Kominers and Sönmez (2016), the types of slots are fixed, whereas in this construction the types of slots depend on the trait distribution of individuals. In other words, C^{mg} can be thought of as an application of the slot-specific choice rules once the types of slots can be determined endogenously, depending on the trait distribution of individuals.

positions. For any trait $t \in \mathcal{T}$ and social category $c \in \mathcal{C}$, let r_t^c denote the number of category-c positions horizontally reserved for trait-t individuals from category-c. In addition, let r_t^o denote the number of open-category positions horizontally reserved for trait-t individuals. These horizontal reservations are provided on a minimum guarantee basis.

For each social category, assume that the sum of horizontal reservations for this category is no more than the number of positions set aside for this category, i.e., for every category $c \in \mathcal{C}$, $\sum_{t \in \mathcal{T}} r_t^c \leq r^c$. An analogous inequality also holds for open positions, i.e., $\sum_{t \in \mathcal{T}} r_t^o \leq r^o$.

4. SCI-VHR Choice Rule & Its Shortcomings

Before we formally define the SCI-VHR choice rule presented in Section 2.1 and mandated throughout India, we show that it is not well-defined when an individual can have more than one trait. That is, its outcome is not uniquely defined, and more specifically it may depend on the details of the adjustment process unspecified under *Anil Kumar Gupta (1995)*. One vertical category is sufficient to illustrate this failure, and hence we will illustrate it through an example with open positions only.

Example 1. Consider an application with four horizontal reservation traits referred to as t_1, t_2, t_3, t_4, and suppose that, based on the allocation of the open positions on the basis of merit in Step 1, the minimum guarantee fails to hold for each of these four categories by one individual. Suppose that the highest merit score individuals among those not chosen in Step 1 are individuals a, b, c, d, and starting with the individual with highest score they are merit-ranked as

$$\sigma(a) > \sigma(b) > \sigma(c) > \sigma(d).$$

Furthermore, each individual qualifies for two of the horizontal reservation traits indicated under the agent as follows.

a	b	c	d
t_1	t_1	t_2	t_3
t_2	t_3	t_4	t_4

For example, individual a qualifies for the horizontal reservation in traits t_1 and t_2. In order to show that the SCI-VHR choice rule is not well-defined, we will carry out the adjustment process with two sequences of traits: $t_1 - t_2 - t_3 - t_4$ and $t_4 - t_3 - t_2 - t_1$.

Case 1 ($t_1 - t_2 - t_3 - t_4$) : Of the four individuals, a and b are the only ones who qualify for horizontal reservation in trait t_1. Having a higher merit score than individual b, individual a will be the first beneficiary of the adjustment process. Observe that individual a qualifies not only for the horizontal reservation in trait t_1, but also for the horizontal reservation in trait t_2. Therefore, with his inclusion the minimum guarantee is satisfied both in trait t_1 and also in trait t_2. Hence, there is no further need for an additional adjustment for trait t_2. Next, consider the adjustment for trait t_3. Of the remaining three individuals, b and d are the only ones who qualify for horizontal reservation in trait t_3. Having a higher merit score than individual d, individual b will be the second beneficiary of the adjustment process. By this point the minimum guarantee is satisfied in traits t_1, t_2, and t_3. Finally, consider the adjustment for trait t_4. Of the remaining two individuals c and d, each one qualifies for horizontal reservation in trait t_4. Having a higher merit score than individual d, individual c will be the third beneficiary of the adjustment process. Since all minimum guarantees are satisfied by this point, no further adjustment is needed and the beneficiaries of the adjustment process are individuals a, b, and c.

Case 2 $(t_4 - t_3 - t_2 - t_1)$: Of the four individuals, c and d are the only ones who qualify for horizontal reservation in trait t_4. Having a higher merit score than individual d, individual c will be the first beneficiary of the adjustment process. Observe that individual c qualifies not only for horizontal reservation in trait t_4, but also for horizontal reservation in trait t_2. Next, consider the adjustment for trait t_3. Of the remaining three individuals, b and d are the only ones who qualify for horizontal reservation in trait t_3. Having a higher merit score than individual d, individual b will be the second beneficiary of the adjustment process. Since individual b qualifies not only for horizontal reservation in trait t_1 but also for horizontal reservation in trait t_3, all four minimum guarantees are satisfied by this point. Hence, no further adjustment is needed, and the beneficiaries of the adjustment process are individuals b and c.

Since two different group of individuals benefit from the adjustment process, the outcome of the SCI-VHR choice rule depends on a detail that is not included in the description of the rule. □

We next provide a formal definition of the SCI-VHR choice rule described in Section 2.1. Since the Supreme Court-mandated choice rule SCI-VHR may fail to be well-defined when individuals are allowed to have multiple traits,[14] we provide a description of this choice rule when individuals have at most one trait.[15] Similarly, we make the same assumption for any result on the SCI-VHR choice rule.

[14] See Example 7 in Appendix C for another irregularity of the Supreme Court-mandated choice function when individuals can have multiple traits.

[15] While in some practical applications candidates are requested to apply for at most one trait of horizontal reservation, (see for example the Madhya Pradesh High Court case Ameer Khan vs State Of M.P. And Ors. (2002), available at `https://indiankanoon.org/doc/1298571/`, last accessed on 02/27/2019), there are also applications where some candidates are eligible for multiple types of horizontal reservation (see for example U.P. State Entrance Examination, Dr. A. P. J. Abdul Kalam Technical University, Uttar Pradesh Information Brochure, available at `https://upsee.nic.in/publicinfo/Handler/FileHandler.ashx?i=File&ii=215&iii=Y`, last accessed on 02/27/2019).

For a set of individuals who are allocated category-c positions, say that trait-t is **oversaturated for c** if the number of trait-t individuals assigned to category-c positions is strictly more than r^c_t. Say that an individual i who is assigned a category-c position is **exposed** if either she does not have a trait or her unique trait $\tau(i)$ is oversaturated for c. In addition, we also use the same terminology for individuals who are allocated open-category positions.

SCI-VHR Choice Rule C^{SCI}

Step 0: Construct the set of open-category eligible individuals I_1 as the union of the set of individuals with r^o highest merit scores and the set of general-category individuals.

Step 1(i): Tentatively choose the individuals with the r^o highest merit scores for the open-category positions.

Step 1(ii): If all open-category horizontal reservations are satisfied for I_1, then proceed to Step 2(i). Otherwise, for each trait t with unsatisfied open-category reservations for I_1, replace

- the lowest merit score exposed chosen individual who has an open-category position with
- the highest merit score unchosen general-category trait-t individual.

Repeat Step 1(ii) until all open-category horizontal reservations are satisfied for I_1.

Step 2(i): For each social category $c \in \mathcal{C}$, let I^c_2 denote the set of category-c individuals who are not chosen yet. Tentatively choose the individuals in I^c_2 with the r^c highest merit scores.

Step 2(ii): For each social category $c \in \mathcal{C}$ and trait $t \in \mathcal{T}$ such that trait-t reservations are not satisfied for I^c_2, replace

- the lowest merit score exposed individual with a category-c position with
- the highest merit score unchosen category-c trait-t individual.

Repeat Step 2(ii) until all category-c horizontal reservations are satisfied for I^c_2.

This process ends in finite time, because, there can only be a finite number of iterations at Steps 1(ii) and 2(ii), and a distinct individual is chosen at each iteration.

For each category, the SCI-VHR choice rule starts by tentatively choosing individuals with the highest merit scores eligible for positions in this category. Then it makes the necessary adjustments for horizontal reservations that are not satisfied. In our next result we provide an equivalent choice rule, presenting the close link between the Supreme Court-mandated choice rule and the single-category merit-maximal choice rule C^{hor} introduced in Section 3.2. This alternative choice rule starts by filling the positions that are reserved horizontally, and consequently it does not require any adjustment steps.

Proposition 3. *Suppose that each individual has at most one trait. Then the SCI-VHR choice rule C^{SCI} is equivalent to the following choice rule.*

Choice Rule $\mathbf{C^{SCI}_{1h}}$
Step 0: *Construct the set of open-category eligible individuals I_1 as the union of the set of individuals with r^o highest merit scores and the set of general-category individuals.*
Step 1: *Choose $C^{hor}(I_1|r^o, (r^o_t)_{t\in\mathcal{T}})$ for the open-category positions.*
Step 2: *For each reserve-eligible social category $c \in \mathcal{C}$, apply $C^{hor}(\cdot|r^c, (r^c_t)_{t\in\mathcal{T}})$ to the remaining category-c individuals and allocate category-c positions to the chosen individuals.*

Since each individual has at most one social category, the order in which reserve-eligible social categories are processed in Step 2 of C^{SCI}_{1h} is immaterial.

Observe that, by Proposition 3, the SCI-VHR choice rule is equivalent to a two-step implementation of the single-category merit-maximal choice rule C^{hor}, however with an important restriction on the set of individuals who are eligible for the allocation of open positions in Step 1. While this restriction is an implication of *Indra Sawhney (1992)*, it is also key to two important shortcomings of the SCI-VHR choice rule that are responsible from numerous lawsuits in India.

4.1. Shortcomings of the SCI-VHR Choice Rule: Failure to Eliminate Justified Envy and Lack of Incentive Compatibility. The SCI-VHR choice rule suffers from two highly visible vulnerabilities that have so far resulted in numerous lawsuits throughout India. We document some of these lawsuits in Section 6.

The source of these vulnerabilities can be easily understood by paying attention to the Step 0 of C^{SCI}_{1h} (or Step 0 of C^{SCI}), the step that determines the eligibility for open-category positions. Individuals with reserve-eligible social categories are ineligible for horizontal adjustments for open-category positions, and they can receive open-category positions by merit only, unless of course, they do not declare their reserve-eligible category and apply as an individual from the general category. While this latter option may give an inferior outcome in most instances, as we present in the next example that is not always the case.

Example 2. Consider a set of individuals with two general-category men m^G_1 and m^G_2, one general-category woman w^G_1, one SC man m^{SC}_3, and one SC woman w^{SC}_2. Suppose that there are two open-category positions and one SC position available. Only one open-category position is reserved for women. Suppose the individuals have the following ranking according to their merit scores:

$$\sigma(m^G_1) > \sigma(m^G_2) > \sigma(m^{SC}_3) > \sigma(w^{SC}_2) > \sigma(w^G_1).$$

When all individuals apply, C^{SCI} works as follows. At Step 1(i), m^G_1 and m^G_2 are tentatively chosen for the open-category positions. The horizontal reservation for women is not satisfied because no woman is allocated a general-category position and there is a rejected general-category woman. Therefore, an adjustment is made at Step 1(ii) and m_2^G is replaced with w_1^G. At Step 2(i), m_3^{SC} is tentatively chosen for the SC position. Since there are no women reservations for SC, no adjustment is made. The set of chosen individuals is $\{m_1^G, w_1^G, m_3^{SC}\}$.

There are two fundamental issues here. The first one is that even though w_2^{SC} has a higher merit score than w_1^G, and w_2^{SC} has a reserve-eligible category while w_1^G does not, w_2^{SC} is rejected while w_1^G is chosen. Woman w_2^{SC} has envy towards w_1^G and her envy is justified because w_2^{SC} has the same horizontal trait as w_1^G, she has a reserve-eligible category while w_1^G does not, and her merit score is higher than that of w_1^G.

The second issue is that if w_2^{SC} does not declare her category SC, then she will be considered a general-category woman and she will be allocated an open-category position at Step 1(ii) because her merit score is higher than that of w_1^G. Therefore, w_2^{SC} has incentives to not declare her caste status and participate as a general-category individual. □

We next formalize these two conceptual issues with the SCI-VHR choice rule. To this end, first consider the following basic fairness property:

Definition 2. *A choice rule C* ***respects inter se merit*** *if, for every $I \subseteq \mathcal{I}$ and $i, j \in I$ with*

(1) $\rho(i) = \rho(j)$,
(2) $\tau(i) = \tau(j)$, and
(3) $\sigma(i) \leq \sigma(j)$

$$i \in C(I) \implies j \in C(I).$$

A choice rule respects *inter se* merit, if an individual with a higher merit score never loses a position to a lower merit score individual with an identical category and set of traits. It is easy to see that the choice rule C^{SCI} respects *inter se* merit, a concept that is mandated by several Supreme Court judgements, and deeply interwoven into modern Indian legal thought.

Given the importance of *inter se* merit in India, one would expect that the following stronger (but even more plausible) principle would also be respected under a Supreme Court-mandated procedure that implements the provisions for positive discrimination.

Definition 3. *A choice rule C* ***eliminates justified envy*** *if, for every $I \subseteq \mathcal{I}$ and $i, j \in I$ with*

(1) $\rho(i) \subseteq \rho(j)$,
(2) $\tau(i) \subseteq \tau(j)$, and
(3) $\sigma(i) \leq \sigma(j)$

$$i \in C(I) \implies j \in C(I).$$

In other words, there is justified envy for a choice rule whenever there exist two individuals i and j such that

(1) either i and j have the same category, or i is a general-category individual (thus lacking any reserve-eligible category),
(2) j has any trait that i has,
(3) j has a higher merit score than i, and
(4) j is rejected from a set of individuals while i is chosen.

Observe that individual j is either from a more disadvantaged category than individual i, or belongs to a more disadvantaged group of citizens possessing additional horizontal traits; and yet she loses a position to individual i despite having a higher merit score. Clearly this is a highly implausible situation. As such, eliminating justified envy is even more important than respecting *inter se* merit, at least in the context of positive discrimination.

Indeed the following quote from the Supreme Court judgement *Rajesh Kumar Daria (2007)* indicates the importance of this fairness principle in India:

> `For example, if there are 200 vacancies and 15% is the vertical reservation for SC and 30% is the horizontal reservation for women, the proper description of the number of posts reserved for SC, should be: ‘‘For SC: 30 posts, of which 9 posts are for women’’. We find that many a time this is wrongly described thus : ‘‘For SC : 21 posts for men and 9 posts for women, in all 30 posts’’. Obviously, there is, and there can be, no reservation category of ‘male’ or ‘men’.`

The last part of this quote indicates that an SC woman cannot loose a position to an SC man of lower merit score, presumably because his set of reservation-qualified categories plus traits is strictly included in hers. Elimination of justified envy is formalization of this principle.

If a choice rule eliminates justified envy, then it also respects *inter se* merit. But even though C^{SCI} respects *inter se* merit, Example 2 shows that it does not eliminate justified envy because w_2^{SC} is rejected while w_1^{G} is chosen when all five individuals apply.

The second issue is that it is against the philosophy of reservation policies that declaring your reserve-eligible category or traits has a potential to hurt you in the allocation process. Before introducing this concept, we define the following auxiliary notion.

An individual **withholds some of her reserve-eligible privileges** if she does not declare either her backward category membership (in case she belongs to one), or some of her traits (or both). For example, a SC individual with a disability can withhold some of her reserve-eligible privileges by not declaring her SC membership or her disability.

Definition 4. *A choice rule C is **incentive compatible** if, for every $I \subseteq \mathcal{I}$ and $i \in I$, if i is chosen from I by withholding some of her reserve-eligible privileges, then i is also chosen by declaring all of her reserve-eligible privileges.*[16]

Incentive compatibility states the following: No individual should be losing a position simply because of declaring all her reserve-eligible privileges (i.e backward class membership or traits).[17] Example 2 shows that C^{SCI} is not incentive compatible because if w_2^{SC} is treated as a general-category female candidate, then she will be chosen when all five candidates apply whereas she is not chosen when she is treated as a SC woman.

5. Alternatives to SCI-VHR Choice Rule

In this section, we provide two modifications of the Supreme Court's choice rule. Each one is not only well-defined regardless of how many traits each individual has, but it also addresses the two fundamental shortcomings identified in Section 4.1. Of the two choice functions, a possible adoption of the first one relies on an "easy to observe" fix, whereas a possible adoption of the latter choice function results in the least "invasive" reform. In that sense, we believe the latter choice function is the more plausible one from a market-design perspective.

The first alternative choice rule is a simple modification of the Supreme Court's choice rule:

Choice Rule $\mathbf{C^{hor}_{2s}}$
Step 1: Apply $C^{hor}(\cdot|r^o, (r^o_t)_{t\in\mathcal{T}})$ to the set of all individuals to allocate the open-category positions.
Step 2: For each social category $c \in \mathcal{C}$, apply $C^{hor}(\cdot|r^c, (r^c_t)_{t\in\mathcal{T}})$ to the remaining category-c individuals and allocate category-c positions to the chosen individuals.

Since the source of the complications of the Supreme Court mandated choice rule was "hidden" in Step 0 of C^{SCI}_{1h} (which restricts access to open-category horizontal adjustments to a subset of the individuals), a straightforward remedy can be obtained by simply deeming every individual eligible for these adjustments, essentially removing Step 0. Under this alternative choice function the single-category merit-maximal choice rule C^{hor} is first applied to all individuals for allocating open-category positions, and then applied for each of the vertical categories for all remaining qualified individuals.

While the description of the choice function C^{hor}_{2s} deviates minimally from the Supreme Court-mandated choice rule C^{SCI}_{1h}, a potential reform based on this alternative may be

[16]Incentive compatibility of a choice rule was first introduced in Aygün and Bó (2016) in the context of affirmative action in Brazilian college admissions.

[17]Incentive compatibility and elimination of justified envy closely relate to each other in the presence of some standard axioms on choice functions. See Appendix A for the formal results.

considered too excessive, because, its outcome can differ than the outcome of C^{hor}_{1h} even in situations where the possible failures of the Supreme Court mandated mechanism is not present. The following example illustrates this possibility.

Example 3. Consider a set of individuals $\mathcal{I}$ with two general-category men m^G_1 and m^G_2, one general-category woman w^G_1, one SC man m^{SC}_3, and one SC woman w^{SC}_2. Suppose that there are two open-category positions and one SC position available. Only one open-category position is reserved for women. Suppose the individuals have the following merit-score ranking:

$$\sigma(m^G_1) > \sigma(m^G_2) > \sigma(w^{SC}_2) > \sigma(m^{SC}_3) > \sigma(w^G_1).$$

In this case, C^{SCI} chooses $\{m^G_1, w^G_1, w^{SC}_2\}$ when every individual applies. This allocation eliminates justified envy, and satisfies all horizontal reservations. It is also easy to see that, no individual who is not chosen can benefit by withholding any reserve-eligible privilege.

However, C^{hor}_{2s} chooses $\{m^G_1, w^{SC}_2, m^{SC}_3\}$ when every individual applies, which is different than $C^{SCI}(\mathcal{I})$. This can be seen as an over adjustment because not only $C^{SCI}(\mathcal{I})$ already eliminates justified envy, but no individual who is not chosen under $C^{SCI}(\mathcal{I})$ can benefit by withholding any reserve-eligible privilege. □

The source of the over adjustment under C^{hor}_{2s} is the following: For any trait $t \in \mathcal{T}$, the choice rule C^{hor}_{2s} considers any trait-t individual from a social category $c \in \mathcal{C}$ for the trait-t open-category positions, prior to her consideration for either vertically reserved positions for category c or a horizontally reserved positions for trait t within category c.

To avoid making over adjustments to the Supreme Court-mandated choice rule we introduce the following alternative, which iteratively applies C^{hor}.

Choice Rule $\mathbf{C^{hor}_{ite}}$

Step 0: Let I_1 be the union of the set of individuals with r^o highest merit scores and the set of general-category individuals.

Step 1: Tentatively choose $C^{hor}(I_1|r^o, (r^o_t)_{t\in\mathcal{T}})$ for the open-category positions.

Step 2: For each category $c \in \mathcal{C}$, apply $C^{hor}(\cdot|r^c, (r^c_t)_{t\in\mathcal{T}})$ to the remaining category-c individuals and allocate category-c positions to the chosen individuals. Stop if no individual is rejected.

Step $2k-1$**,** $k > 1$**:** Apply $C^{hor}(\cdot|r^o, (r^o_t)_{t\in\mathcal{T}})$ to the union of the set of individuals who are tentatively allocated the open-category positions and the set of individuals without a position and allocate the open-open category positions to the chosen individuals. Stop if the set of individuals without a position remains the same.

Step $2k$**,** $k > 1$**:** For each category $c \in \mathcal{C}$, apply $C^{hor}(\cdot|r^c, (r^c_t)_{t\in\mathcal{T}})$ to the set of category-c individuals who are not allocated an open-category position in Step

$2k-1$ and allocate category-c positions to the chosen individuals. Stop if the set of individuals without a position remains the same.

This choice rule terminates in finite time because for a category either (1) the set of chosen individuals remains the same at a step and the rule terminates or (2) a different set of individuals is chosen and more horizontally reserved seats are filled or (3) a different set of individuals is chosen and the chosen set is ranked strictly higher by the dominance relation than the set chosen at the previous step.

Observe that Steps 0-2 of the choice rule C^{hor}_{ite} coincides with Steps 0-2 of the choice rule C^{SCI}_{1h}, where the latter is equivalent to C^{SCI} by Proposition 3 when each individual has at most one trait. The following example illustrates the working of C^{hor}_{ite}.[18]

Example 4. Consider a set $\mathcal{I}$ of 11 individuals with three general-category men m^G_1, m^G_2, m^G_3, two general-category women w^G_1, w^G_2, two SC men m^{SC}_4, m^{SC}_5, and three SC women w^{SC}_4, w^{SC}_5, w^{SC}_6. Suppose these individuals have the following merit-score ranking:

$$\sigma(m^G_1) > \sigma(m^{SC}_4) > \sigma(m^G_2) > \sigma(m^G_3) > \sigma(w^{SC}_3) > \sigma(w^{SC}_4) > \sigma(m^{SC}_5) > \sigma(w^{SC}_5) > \sigma(w^G_1) > \sigma(w^G_2) > \sigma(w^{SC}_6).$$

There are four open-category positions, three of which are reserved for women. In addition, there are two SC positions, one of which is reserved for women.

When all individuals apply C^{SCI} works as follows. At Step 0, the set of open-category eligible individuals is constructed as

$$I_1 = \{m^G_1, m^{SC}_4, m^G_2, m^G_3, w^G_1, w^G_2\}.$$

At Step 1, $C^{hor}(I_1|r^o, r^o_w) = \{m^G_1, m^{SC}_4, w^G_1, w^G_2\}$ is chosen for the open-category positions. At Step 2, the unassigned SC individuals are w^{SC}_3, w^{SC}_4, m^{SC}_5, w^{SC}_5, and w^{SC}_6. Choice rule $C^{hor}(\cdot|r^{SC}, r^{SC}_w)$ is applied to this set and individuals in $\{w^{SC}_3, w^{SC}_4\}$ are given the SC positions. Therefore,

$$C^{SCI}(\mathcal{I}) = \{m^G_1, m^{SC}_4, w^{SC}_3, w^{SC}_4, w^G_1, w^G_2\}.$$

Next we run the algorithm for C^{hor}_{ite} when all individuals apply. Steps 0-2 of C^{hor}_{ite} are the same as the corresponding steps of C^{SCI}, although the allocation obtained at the end of Step 2 is tentative under C^{hor}_{ite}, different than C^{SCI} where it was final. At Step 3, all individuals except those who are tentatively assigned SC positions at Step 2 are considered for the open-category positions, which is

$$\{m^G_1, m^{SC}_4, m^G_2, m^G_3, m^{SC}_5, w^{SC}_5, w^G_1, w^G_2, w^{SC}_6\}.$$

[18] Example 4 is constructed so that it not only illustrates the choice function C^{hor}_{ite}, but it also demonstrates the necessity of the iterative aspect of the choice function while at the same time producing an outcome that differs from both C^{SCI} and C^{hor}_{2s}.

Choice rule $C^{hor}(\cdot|r^o, r^o_w)$ is applied to this set and $\{m^G_1, w^{SC}_5, w^G_1, w^G_2\}$ is tentatively chosen for the open-category positions. At Step 4, all SC individuals who are not assigned an open-category positions are considered for the SC positions: m^{SC}_4, w^{SC}_3, w^{SC}_4, m^{SC}_5, w^{SC}_6. Choice rule $C^{hor}(\cdot|r^{SC}, r^{SC}_w)$ is applied to this set and individuals in $\{m^{SC}_4, w^{SC}_3\}$ are tentatively assigned SC positions. At Step 5, all individuals except those who are tentatively assigned SC positions at Step 4 are considered for the open-category positions, which is

$$\{m^G_1, m^G_2, m^G_3, w^{SC}_4, m^{SC}_5, w^{SC}_5, w^G_1, w^G_2, w^{SC}_6\}.$$

Choice rule $C^{hor}(\cdot|r^o, r^o_w)$ is applied to this set and $\{m^G_1, w^{SC}_4, w^{SC}_5, w^G_1\}$ is tentatively chosen for the open-category positions. At Step 6, all SC individuals not assigned to an open-category position, m^{SC}_4, w^{SC}_3, m^{SC}_5, w^{SC}_6, are considered for the SC positions. Choice rule $C^{hor}(\cdot|r^{SC}, r^{SC}_w)$ is applied to this set and individuals in $\{m^{SC}_4, w^{SC}_3\}$ are tentatively assigned SC positions. Since the set of unassigned individuals does not change at Step 6, the algorithm terminates and, therefore,

$$C^{hor}_{ite}(\mathcal{I}) = \{m^G_1, m^{SC}_4, w^{SC}_3, w^{SC}_4, w^{SC}_5, w^G_1\}.$$

Finally, we also consider choice rule C^{hor}_{2s}. At the first step, all individuals are considered for the open-category positions and $C^{hor}(\mathcal{I}|r^o, r^o_w) = \{m^G_1, w^{SC}_3, w^{SC}_4, w^{SC}_5\}$ is selected. At Step 2, all unassigned SC individuals m^{SC}_4, m^{SC}_5, w^{SC}_6 are considered for SC positions and $C^{hor}(\cdot|r^{SC}, r^{SC}_w)$ is applied to this set to select $\{m^{SC}_4, w^{SC}_6\}$ for the SC positions. Hence,

$$C^{hor}_{2s}(\mathcal{I}) = \{m^G_1, m^{SC}_4, w^{SC}_3, w^{SC}_4, w^{SC}_5, w^{SC}_6\}.$$

□

We are ready to present our results for the two alternative choice functions. As promised, both of them escape the shortcomings of the Supreme Court-mandated choice rule presented in Section 4.1.

Proposition 4. *Both C^{hor}_{2s} and C^{hor}_{ite} eliminate justified envy and are incentive compatible.*

We next show that between the choice rule C^{SCI} and its two alternatives C^{hor}_{2s}, C^{hor}_{ite},

(1) C^{SCI} produces the best and C^{hor}_{2s} produces the worst outcome for general-category individuals, whereas
(2) in terms of the total number of positions assigned to members of social categories, C^{hor}_{2s} produces the best outcome and C^{SCI} produces the worst outcome.

Proposition 5. *Suppose that each individual has at most one trait. For every $I \subseteq \mathcal{I}$,*

$$C^{SCI}(I) \cap I^G \supseteq C^{hor}_{ite}(I) \cap I^G \supseteq C^{hor}_{2s}(I) \cap I^G$$

and

$$\left|C^{SCI}(I) \cap I^R\right| \leq \left|C^{hor}_{ite}(I) \cap I^R\right| \leq \left|C^{hor}_{2s}(I) \cap I^R\right|.$$

Proposition 5 shows that, of the two alternative choice function, the outcome of C^{hor}_{ite} is "closer" to the outcome of C^{SCI}. In our main result, we show that a potential reform based on C^{hor}_{ite} can be interpreted as one based on a "minimal deviation" from C^{SCI} while correcting its shortcomings.

Theorem 1. *Suppose that each individual has at most one trait. Let $I \subseteq \mathcal{I}$ be a set of individuals such that $C^{SCI}(I)$ satisfies open-category horizontal reservations for I. Then*

(1) if $C^{SCI}(I)$ eliminates justified envy, then $C^{hor}_{ite}(I) = C^{SCI}(I)$, and
(2) if $C^{SCI}(I)$ fails to eliminate justified envy, then assuming that there are sufficiently many individuals to fill the positions at each vertical category and that there is only one horizontal trait,

$$\left|C^{SCI}(I) \setminus C^{hor}_{ite}(I)\right| \leq \left|C^{SCI}(I) \setminus C(I)\right|$$

for any choice rule C where $C(I)$ eliminates justified envy.

The following example illustrates that Part 2 of Theorem 1 does not hold when there are two traits or more.

Example 5. Consider a set of individuals $\mathcal{I}$ with one general-category woman w^G_1, one general-category disabled individual d^G_1, two SC women w^{SC}_2, w^{SC}_3, and one disabled SC individual d^{SC}_2. Suppose that there are two open-category positions and one SC position available. One open-category position is reserved for women and one for disabled individuals. Suppose the individuals have the following merit-score ranking:

$$\sigma(w^{SC}_2) > \sigma(d^{SC}_2) > \sigma(w^{SC}_3) > \sigma(w^G_1) > \sigma(d^G_1).$$

When all individuals apply, C^{SCI} allocates the positions to $\{w^G_1, d^G_1, w^{SC}_2\}$. This allocation satisfies open-category horizontal reservations because there is one woman, w^G_1, and one disabled individual, d^G_1, who gets an open-category position. However, there is justified envy because d^{SC}_2 is a disabled SC individual who is unassigned whereas d^G_1 is a disabled general-category individual with a position who has a lower merit score than d^{SC}_2. Likewise, w^{SC}_3 is unassigned whereas w^G_1 is assigned a position.

The first two steps of C^{hor}_{ite} are the same as the corresponding steps of C^{SCI}. At Step 3 of C^{hor}_{ite}, individuals in $\{w^G_1, d^G_1\}$ lose their positions to individuals in $\{w^{SC}_3, d^{SC}_2\}$. The algorithm terminates at Step 4 and C^{hor}_{ite} produces $\{w^{SC}_3, d^{SC}_2, w^{SC}_2\}$. Therefore, $\left|C^{SCI}(\mathcal{I}) \setminus C^{hor}_{ite}(\mathcal{I})\right| = \left|\{w^G_1, d^G_1\}\right| = 2$.

Now consider the following choice rule C, which removes w^G_1 from $C^{SCI}(\mathcal{I})$ and assigns that open-category position to w^{SC}_2. In addition, C assigns the vacated SC position to d_2^{SC}. Therefore, $C(\mathcal{I}) = \{w_2^{SC}, d_1^G, d_2^{SC}\}$. This outcome eliminates justified envy and, furthermore, $\left|C^{SCI}(\mathcal{I}) \setminus C(\mathcal{I})\right| = \left|\{w_1^G\}\right| = 1 < \left|C^{SCI}(\mathcal{I}) \setminus C^{hor}_{ite}(\mathcal{I})\right|$. □

6. Litigations on the Supreme Court-Mandated Choice Rule

As we have presented in Section 4.1, the SCI-VHR choice rule allows for justified envy. Moreover, it fails to be incentive compatible due to backward class candidates losing their access to horizontally reserved positions in the open category by declaring their backward class status.

The failure of SCI-VHR choice rule to eliminate justified envy is fairly straightforward to observe. All it takes is a rejected backward class candidate to realize that her merit score is higher than an accepted general-category candidate, even though she has all the horizontal traits the admitted candidate does. Since the primary role of the reservation policy is positive discrimination for candidates with more vulnerable backgrounds, this situation is very counterintuitive, and it often results in legal action.

Focusing on complications caused either by the presence of justified envy or the lack of incentive compatibility, we next present several court cases to illustrate that these shortcomings significantly handicap the system in India.

6.1. High Court Cases Related to Justified Envy. The failure of SCI-VHR choice rule to eliminate justified envy has resulted in numerous court cases throughout India, and since the presence of justified envy in the system is highly implausible, these legal challenges often result in controversial rulings. In addition, there are also cases where authorities who implement a better-behaved version of the choice rule, one that does not suffer from this shortcoming, are nonetheless challenged in court, on the basis that their adopted choice rules differ from the one mandated by the Supreme Court. These court cases are not restricted to lower courts, and include several cases argued in state high courts. Even at the level of state high courts, the judgements on this issue are highly inconsistent, mostly because of the confusion caused by the possibility of justified envy under the SCI-VHR choice rule. We next present several representative cases from High Courts of five states:

(1) *Rajeshwari vs State (Panchayati Raj Dep) Ors, 15 March, 2013*, Rajasthan High Court.[19] This case combines 120 petitions against the State of Rajasthan where the petitioners seek legal action on the basis that reserve category women are allowed to benefit from open-category horizontally reserved positions for women. The high court rules that the state is at fault, and it must abandon its choice rule,

[19]The case is available at `https://indiankanoon.org/doc/128221069/` (last accessed on 03/07/2019).

adopting the one mandated by the Supreme Court. The following quote is from a story published in The Times of India covering this court case:[20]

> In a judgment that would affect all recruitments in the state government, the Rajasthan high court has ruled that posts reserved for women in the open/general category cannot be filled with women from reserved categories even if the latter are placed higher on the merit list...
> Women candidates who contested for different positions in at least three government departments, including the panchayati raj, education and medical, last year had challenged the government move to allow ''migration'' of reserved category women to fill the open category seats. The positions applied for included that of teachers Grade-II and III, school lecturers, headmasters and pharmacists.

Ironically, while the high court's decision is correct, it also means that the better-behaved version of the choice rule has to be abandoned throughout the state.

(2) *Ashish Kumar Pandey And 24 Others vs State Of U.P. And 29 Others on 16 March, 2016*, Allahabad High Court.[21] In a case that mimics the aforementioned Rajasthan High Court case, this lawsuit was brought to Allahabad High Court by 25 petitioners, disputing the mechanism employed by the State of Uttar Pradesh—the most populous state in India with more than 200 million residents—to apply the provisions of horizontal reservations in their allocation of more than 4000 civil police and platoon commander positions. Of these positions, 27%, 21%, 2% are each vertically reserved for backward classes OBC, SC, and ST, respectively, and 20%, 5%, and 2% are each horizontally reserved for women, ex-servicemen, and dependents of freedom fighters, respectively. While only 19 women are selected for open-category positions based on their merit scores, the total number of female candidates is less than even the number of open-category horizontally reserved positions for women, and as such all remaining women are selected. However, instead of assigning them positions from their respective backward class categories (as it is mandated by the Supreme Court), all of them are assigned positions from the open category. Similarly, backward class candidates are deemed eligible to use horizontal reservations for dependents of freedom fighters and ex-serviceman as well. The counsel for the petitioners argues that not only did the State of U.P. make an error in their implementation of horizontal reservations, but also that the error was intentional. The following quote is from the court case:

[20]The Times of India story is available at https://timesofindia.indiatimes.com/city/jaipur/Womens-seats-on-open-merit-cant-be-filled-from-SC/ST-quota-High-court/articleshow/19101277.cms (last accessed on 03/07/2019).

[21]The case is available at https://indiankanoon.org/doc/74817661/ (last accessed on 03/07/2019).

> Per contra, learned counsel appearing for the petitioners would submit that fallacy was committed by the Board deliberately, and with malafide intention to deprive the meritorious candidates their rightful placement in the open category. The candidates seeking horizontal reservations belonging to OBC and SC category were wrongly adjusted in the open category, whereas, they ought to have been adjusted in their quota provided in respective social category. The action of the Board is not only motivated, but purports to take forward the unwritten agenda of the State Government to accommodate as many number of OBC/SC candidates in the open category.

The judge of the case sides with the petitioners, and rules that the State of Uttar Pradesh must correct their erroneous application of the provisions of horizontal reservations. The judge further emphasizes that the State has played foul, stating:

> There is merit in the submission of the learned counsel for the petitioners that the conduct of the members of the Board appears not only mischievous but motivated to achieve a calculated agenda by deliberately keeping meritorious candidates out of the select list. The Board and the officials involved in the recruitment process were fully aware of the principle of horizontal reservations enshrined in Act, 1993 and Government Orders which were being followed by them in previous selections of SICP and PC (PAC), but in the present selection they chose to adopt a principle against their own Government Orders and the statutory provisions which were binding upon them...
> I am constrained to hold that both the State and the Board have played fraud on the principles enshrined in the Constitution with regard to public appointment.

What is especially surprising is, despite the heavy tone of this judgement, the State goes on to appeal in another Allahabad High Court case *State Of U.P. And 2 Ors. vs Ashish Kumar Pandey And 58 Ors, 29 July, 2016*,[22] in an effort to continue using its preferred method for implementing horizontal reservations. Perhaps unsurprisingly, this appeal was denied by the High Court.

This particular case clearly illustrates that there is a strong resistance in at least some of the states to implementing the provisions of horizontal reservations in their Supreme Court-mandated form. While this resistance most likely reflects the

[22]The case is available at `https://indiankanoon.org/doc/71146861/` (last accessed on 03/07/2019).

political nature of this debate, the arguments of the counsel for the State to maintain their preferred mechanism to implement the provisions of horizontal reservations are mostly based on the presence of justified envy under the Supreme Court-mandated version. The following quote from the appeal illustrates that this was the main argument used in their defense:

> `The arguments that have been advanced on behalf of State and private appellant with all vehemence that women candidates irrespective of their social class i.e. SC/ST/OBC are entitled to make place for themselves in an open category on their inter-se merit clearly gives an impression to us that State of U.P and its agents/servants and even the private appellants are totally unaware of the distinction that has been time and again reiterated in between vertical reservation and horizontal reservation and the way and manner in which the provision has to be pressed and brought into play.`

(3) *Asha Ramnath Gholap vs President, District Selection Committee & Ors. on March 3rd, 2016*, Bombay High Court.[23] In this case, there are 23 pharmacist positions to be allocated; 13 of these positions are vertically reserved for backward classes and the remaining ten are open for all candidates. In the open category, eight of the ten positions are horizontally reserved for various groups, including three for women. The petitioner, Asha Ramnath Gholap, is an SC woman, and while there is one vertically reserved position for SC candidates, there is no horizontally reserved position for SC women. Under the SCI-VHR choice rule, she is not eligible for any of the horizontally reserved women positions at the open category. Nevertheless, she brings her case to the Bombay High Court based on an instance of justified envy, described in the court records as follows:

> `It is the contention of the petitioner that Respondent Nos. 4 & 5 have received less marks than the petitioner and as such, both were not liable to be selected. The petitioner has, therefore, approached this court by invoking the writ jurisdiction of this court under Article 226 of the Constitution of India, seeking quashment of the select list to the extent it contains the names of Respondent Nos.4 and 5 against the seats reserved for the candidates belonging to open female category.`

There is no merit to this argument, because the choice rule mandated by the Supreme Court allows for justified envy. However, the judges sided with the petitioner on the basis that a candidate cannot be denied a position from the open

[23]The case is available at `https://indiankanoon.org/doc/178693513/` (last accessed on 03/08/2019).

category based on her backward class membership, essentially ruling out the possibility of justified envy under a Supreme Court-mandated choice rule, which is designed to allow for positive discrimination for the vulnerable groups in the society.[24] Their justification is given in the court records as follows:

> We find the argument advanced as above to be fallacious. Once it is held that general category or open category takes in its sweep all candidates belonging to all categories irrespective of their caste, class or community or tribe, it is irrelevant whether the reservation provided is vertical or horizontal. There cannot be two interpretations of the words 'open category' ...

(4) *Uday Sisode vs Home Department (Police) on 24 October, 2017*, Madhya Pradesh High Court.[25] In another case parallel to that at Bombay High Court, the judges of Madhya Pradesh High Court issued a questionable decision by siding with a petitioner who filed this lawsuit based on another instance of justified envy.

(5) *Smt. Megha Shetty vs State Of Raj. & Anr on 26 July, 2013*, Rajasthan High Court.[26] In contrast to *Asha Ramnath Gholap (2016)* and *Uday Sisode (2017)* where the judges have been erroneous siding with petitioners whose lawsuits are based on instances of justified envy, in this case a petitioner who is a member of the general category seeks legal action against the State on the basis that several horizontally reserved open-category women positions are allocated to women from OBC who are not eligible for these positions (unless they receive it without invoking the benefits of horizontal reservation). While all these OBC women have higher merit scores than the petitioner and the State have apparently used a better behaved procedure, the petitioner's case has merit because the Supreme Court-mandated procedure allows for justified envy in those situations. In an earlier lawsuit, the petitioner's lawsuit was already declined by a single judge of the same court based on an erroneous interpretation of *Indra Sawhney (1992)*. The petitioner subsequently appeals this erroneous decision and brings the case to a larger bench of the same court.

[24]In a very similar Bombay High Court case *Rajani Shaileshkumar Khobragade ... vs The State Of Maharashtra And ... on 31 March, 2017* where the petitioner filed a lawsuit based on another instance of justified envy, the judges of the same high court dismissed the petition. This case is available at `https://indiankanoon.org/doc/7250640/`, last accessed on 03/09/2019. Indeed, there seem to be several conflicting decisions at the Bombay High Court on this very issue, including a series of cases reported in a July 18, 2018 dated *The Times of India* story "MPSC won't issue job letters till HC hears plea on quota issue" available at `https://timesofindia.indiatimes.com/city/aurangabad/mpsc-wont-issue-job-letters-till-hc-hears-plea-on-quota-issue/articleshow/65029505.cms` (last accessed on 03/09/2019).

[25]The case is available at `https://indiankanoon.org/doc/196750337/` (last accessed on 03/08/2019).

[26]The case is available at `https://indiankanoon.org/doc/78343251/` (last accessed on 10/08/2019).

However, the three judges side with the earlier judgement, thus erroneously dismissing the appeal. Their decision is justified as follows:

> The outstanding and important feature to be noticed is that it is not the case of the appellant-petitioner that she has obtained more marks than those 8 OBC (Woman) candidates, who have been appointed against the posts meant for General Category (Woman), inasmuch as, while the appellant is at Serial No.184 in the merit list, the last OBC (Woman) appointed is at Serial No.125 in the merit list. The controversy raised by the appellant is required to be examined in the context and backdrop of these significant factual aspects.

As seen from this argument, many judges have difficulty perceiving that the Supreme Court-mandated procedure could possibly allow for justified envy.

(6) *Mukta Purohit & Ors vs State & Ors on 12 April, 2018*, Rajasthan High Court.[27] In a case that mimics *Smt. Megha Shetty (2013)*, judges of the Rajasthan High Court erroneously dismiss a petition filed against the state that allowed horizontally reserved open-category women positions to be allocated to women from reserved categories who are not eligible for these positions. Indeed *Smt. Megha Shetty (2013)* is used as a precedent in this judgement.

(7) *Arpita Sahu vs The State Of Madhya Pradesh on 21 August, 2012* Madhya Pradesh High Court.[28]

The petitioner files a lawsuit based on an instance of justified envy, however in contrast to *Asha Ramnath Gholap (2016)* and *Uday Sisode (2017)*, the judges have correctly dismissed the petition in this case.

(8) *Mamta Bisht vs State of Uttaranchal And Others, 26 October, 2005*, Uttarakhand High Court.[29][30] In this case, there are 42 civil judge positions to be allocated in Uttaranchal. The petitioner, Mamta Bisht, is eligible for horizontally reserved positions for Uttaranchal women, but her merit score is not high enough to secure a position either through the open category, or from Uttaranchal-women category. She files a petition based on the following argument: The merit score of the lowest score candidate who secured a position in the open category is lower than the score of Neetu Joshi, who is the highest merit score candidate who benefitted from horizontally reserved positions for Uttaranchal women.[31] The petitioner argues that,

[27]The case is available at https://indiankanoon.org/doc/126738191/ (last accessed on 10/10/2019).

[28]The case is available at https://indiankanoon.org/doc/102792215/ (last accessed on 10/10/2019).

[29]The case is available at https://www.casemine.com/judgement/in/56b494fb607dba348f01036a (last accessed on 03/07/2019).

[30]The name of this state was changed from Uttaranchal to Uttarakhand in 2007. We use Uttaranchal in our discussion because this is the name used in the court case.

[31]This situation was possible due to an additional position horizontally reserved for ex-military personnel.

Neetu Joshi has to receive the last open-category position due to the fact that her merit score is higher than that of the lowest score candidate admitted for one of these positions, and that candidate, having the highest merit score among remaining Uttaranchal-women candidates, has to receive the horizontally reserved position Neetu Joshi no longer needs to occupy. The high court allows her petition, and in its decision grants her a position based on the following justification:

> In view of above, Neetu Joshi, (SI. No. 9, Roll No. 12320) has wrongly been counted by respondent No. 3 / Commission against five seats reserved for Uttaranchal Women General Category as she has competed on her own merit as general candidates and as 5th candidate the petitioner should have been counted for Uttaranchal Women General Category seats.

This erroneous high court judgement was later overruled by the Supreme Court in their civil appellate case *Public Service ... vs Mamta Bisht And Ors on 3 June, 2010*,[32] but not before setting a precedent for several subsequent lawsuits.

6.2. Wrongful Implementation and Possible Misconduct. It is bad enough that the Supreme Court-mandated choice rule is not incentive compatible, forcing some candidates to choose between declaring their social reservation-eligible backward class status and their special reservation-eligible horizontal traits. To make matters worse, in some cases candidates are denied access to open-category horizontally reserved positions even when they do not submit their backward class status, giving up their eligibility for vertically reserved positions for their reserve-eligible class. Therefore, even when the candidate applies for a position as a general-category candidate, the central planner processes the application as if the backward class status was claimed, denying the candidate's eligibility for open-category horizontally reserved positions for her trait. The central planners are able to do this, because last names in India are, to a large extent, indicative of a caste membership. This type of misconduct seems to be fairly widespread, and it is the main cause of the lawsuit in each of the following cases:

(1) *Shilpa Sahebrao Kadam And Another vs The State Of Maharashtra And ... on 8 August, 2019*, Bombay High Court.[33]
(2) *Vinod Kadubal Rathod And Another vs Maharashtra State Electricity ... on 17 February, 2017*, Bombay High Court.[34]

[32] The case is available at `https://indiankanoon.org/doc/518824/` (last accessed on 03/07/2019).
[33] The case is available at `https://indiankanoon.org/doc/89017459/` (last accessed on 03/09/2019).
[34] The case is available at `https://indiankanoon.org/doc/162611497/` (last accessed on 03/09/2019).

(3) Original Applications 1007, 1052, 1056, 1057 & 1070/2017 dated 29.11.2017, Maharashtra Administrative Tribunal, Mumbai Bench.[35]

(4) Original Application 529 of 2017 dated 28.09.2017, Maharashtra Administrative Tribunal, Mumbai Bench.[36]

(5) Original Applications 944, 945 & 220/2017 dated 20.07.2018, Maharashtra Administrative Tribunal, Mumbai Bench at Aurangabad.[37]

Indeed, this type of misconduct seems to be intentional and systematic in some juristictions. The following statement is from *Shilpa Sahebrao Kadam (2019)*:

```
According to Respondent - Maharashtra Public Service Commission, in
view of the Circular dated 13.08.2014, only the candidates belonging
to open (Non-reserved) category can be considered for open horizontally
reserved posts meaning thereby, the reserved category candidates cannot
be considered for open horizontally reserved post.  Reference is made to
a communication issued by the Additional Chief Secretary (Service) of the
State of Maharashtra dated 26.07.2017, whereunder it is prescribed that
a female candidate belonging to any reserved category, even if tenders
application form seeking employment as an open category candidate, the name
of such candidate shall not be recommended for employment against a open
category seat.
```

Moreover, not all decisions in these lawsuits are made in accordance with the Supreme Court-mandated procedure. This is the case both for the first lawsuit and the last one listed above. For example, in the last lawsuit given above, two petitioners each applied for a position without declaring their backward class membership, with an intention to benefit from open-category horizontal reservations. Following their application, these petitioners were requested to provide their school leaving certificates, which provided information on their backward class status. Upon receiving this information, the petitioners were declined eligibility for the provisions of open-category horizontal reservations, even though they never claimed the benefits of backward class vertical reservations. Hence, they filed the fourth lawsuit given above. Remarkably, their petition was declined on the basis of their backward class membership. Here we have a case where the authorities not only go to great lengths to obtain the backward class membership of the candidates, and wrongfully decline their eligibility for special horizontal reservations, but they also manage to get their lawsuits dismissed.

[35]The case is available at `https://mat.maharashtra.gov.in/Site/Upload/Pdf/O.A%201007.17%20and%20ors%20DB,%2029.11.17,%20Chairman.PDF` (last accessed on 03/09/2019).

[36]The case is available at `https://mat.maharashtra.gov.in/Site/Upload/Pdf/O.A%20529.17%20Appointment%20challenged,%20DB.0917.PDF` (last accessed on 03/09/2019).

[37]The case is available at `https://mat.maharashtra.gov.in/Site/Upload/Pdf/944%20945%20&%20220%20of%202017.pdf` (last accessed at 03/09/2019).

The mishandling of this case is consistent with the concerns indicated in the February 2006 issue of *The Inter-Regional Inequality Facility* policy brief:[38]

> Another issue relates to the access of SCs and STs to the institutions of justice in seeking protection against discrimination. Studies indicate that SCs and STs are generally faced with insurmountable obstacles in their efforts to seek justice in the event of discrimination. The official statistics and primary survey data bring out this character of justice institutions. The data on Civil Rights cases, for example, shows that only 1.6% of the total cases registered in 1991 were convicted, and that this had fallen to 0.9% in 2000.

6.3. Loss of Access to Horizontal Reservations without any Access to Vertical Reservations. The main justification offered in various Supreme Court cases for denying backward class members the provisions of horizontal reservations for open-category positions is avoiding a situation where an excessive number of positions are reserved for members of these classes. In several cases, however, members of these classes are denied access to horizontally reserved positions even when their reserve-eligible vertical category is not earmarked for those positions. This is the case in the following two court cases:

(1) *Tejaswini Raghunath Galande v. The Chairman, Maharashtra Public Service Commission and Ors. on 23 January 2019*, Writ Petition Nos. 5397 of 2016 & 5396 of 2016, High Court of Judicature at Bombay.[39]
(2) Original Application No. 662/2016 dated 05.12.2017, Maharashtra Administrative Tribunal, Mumbai.[40]

In both of the above cases, while both petitioners declared their backward class status, there was no position vertically reserved for their class. Yet they both lost access to horizontally reserved positions in the open category for their traits. In the first case, the petitioners' lawsuit to benefit from horizontal reservations was initially declined by a lower court, resulting in the appeal at the High Court. The lower court's decision was overruled in the High Court, and her request was granted. The second petitioner's similar request was declined by the Maharashtra Administrative Tribunal. What is more worrisome in the second case is that initially three positions were announced to be vertically reserved for the petitioner's backward class, but after her application these positions were withdrawn. Therefore, the candidate declared her backward class status, giving up her

[38]The policy brief is available at `https://www.odi.org/sites/odi.org.uk/files/odi-assets/publications-opinion-files/4080.pdf` (last accessed 03/09/2019).

[39]The case is available at `https://www.casemine.com/judgement/in/5c713d919eff4312dfbb5900` (last accessed on 03/09/2019).

[40]The case is available at `https://mat.maharashtra.gov.in/Site/Upload/Pdf/O.A.662%20of%202016.pdf` (last accessed on 03/09/2019).

eligibility for several horizontally reserved women positions at the open category, presumably to gain access to vertically reserved positions for her backwards class, only to learn that she had given up her eligibility for nothing.

7. The Implications of 103rd Amendment of the Constitution of India

In a highly debated reform on the reservation system, the *One Hundred and Third Amendment of the Constitution of India* provides 10% reservation to the economically weaker sections (EWS) in the general category.[41] A government memorandum dated 01/31/2019 specifies these new provisions as a vertical reservation:[42]

```
7.  ADJUSTMENT AGAINST UNRESERVED VACANCIES:
  A person belonging to EWS cannot be denied the right to compete for
appointment against an unreserved vacancy.  Persons belonging to EWS who
are selected on the basis of merit and not on account of reservation are
not to be counted towards the quota meant for reservation.
```

The One Hundred and Third Amendment was immediately challenged at the Supreme Court, and as of October 2019 the case is still pending.[43] Despite being challenged at the Supreme Court, the EWS reservation has already been adopted by federal institutions throughout India as well as by most states at their state-run public institutions. If the One Hundred and Third Amendment survives the Supreme Court challenge, it will likely amplify the legal challenges formalized in Section 4.1 and documented in Section 6. Especially in states with a strong presence of horizontal reservation (such as those with 30-35% women reservation), legal challenges based on justified envy may become the norm rather than an exception if the amendment survives. That is because, any candidate who applies both for the EWS reservation and any horizontal reservation will loose access to open-category horizontally reserved positions under the Supreme Court-mandated choice rule.

It is estimated that, around 26% of the population in India does not belong to the Other Backward Classes (OBC), Scheduled Castes (SC) and Scheduled Tribes (ST) categories.[44] Therefore, prior to the new amendment, about 26% of the population used to belong to

[41]The bill of the *One Hundred and Third Amendment of the Constitution of India* was introduced in the Lok Sabha—the lower house of the Parliament of India—on 01/08/2019 as the Constitution (One Hundred and Twenty-fourth Amendment) Bill, 2019. The bill was passed by the Lok Sabha on 01/09/2019, by the Rajya Sabha—the upper house of the Parliament of India—on 01/10/2019, and came into effect on 01/14/2019.

[42]See the Government of India Ministry of Personnel, Public Grievances & Pensions Department of Personnel & Training memorandum No. 36039/1/2019 on Reservation for Economically Weaker Sections (EWSs) in direct recruitment in civil posts and services in the Government of India. This memorandum is available at `https://dopt.gov.in/sites/default/files/ewsf28fT.PDF`, last accessed 04/14/2019.

[43]See `https://www.scobserver.in/court-case/reservations-for-economically-weaker-sections` for the pending Supreme Court case *Youth for Equality v. Union of India*.

[44]See the 01/07/2017-dated *Hindustan Times* story "Quota for economically weak in general category could benefit 190 mn," which is available at `https://www.hindustantimes.com/`

the general category. While the amendment is intended for the economically weaker sections of the general category, according to most estimates more than 80% of the members of this group satisfy the eligibility criteria for the EWS reservation.[45] This means, with the introduction of the EWS reservation, the fraction of the population who are ineligible for any vertical reservation reduces to roughly 5-6% of the population. Therefore, the "new general category," those members of the society who are ineligible for any vertical reservations, shrinks to approximately 5-6% of the whole population.[46] A key implication of this observation is the following: Unless the Supreme Court-mandated choice rule is amended in a manner addressing the shortcomings presented in Section 4.1, only this "elite" 5-6% of the population qualifies for adjustments for open-category horizontal reservation. For example, consider a woman who qualifies for the 10% EWS reservation. In a state with 30% women reservation, she will now be qualified for the horizontally reserved EWS-women positions which makes 3% of all positions. However, on the other hand she will loose access to open positions that are horizontally reserved for women which is 12% of all positions. This anomaly will likely increase the instances of justified envy considerably throughout India, especially in states with extensive use of horizontal reservation, such as Bihar with 35%, Andhra Pradesh with $33\frac{1}{3}$%, and Madhya Pradesh, Rajasthan, Uttarakhand, Chhattisgarh, Sikkim with 30% women reservation.

8. Conclusion

In a highly politicized reform that was undertaken just a few months prior to the 2019 General Elections in India, a ceiling of 10% vertical reservations was granted to Economically Weaker Sections of the General Category. Many have argued that this reform gave an advantage to the Prime Minister Narendra Modi's party BJP in the elections. Subsequently, several petitions have challenged the constitutionality of the 103rd Amendment of the Constitution of India, in a pending Supreme Court case *Youth for Equality v. Union*

india-news/quota-for-economically-weak-in-general-category-could-benefit-190-mn/story-6vvfGmXBohmLrCYkgM1NYJ.html, last accessed on 04/14/2019.

[45]See the 01/08/2019 dated *Business Today* story "In-depth: Who is eligible for the new reservation quota for general category?" which is available at https://www.businesstoday.in/current/economy-politics/in-depth-who-is-eligible-for-the-new-reservation-quota-for-general-category/story/308062.html, and the 01/28/2019 dated *The Indian Express* story "Whose quota is it anyway? Eligibility criteria for reservation for economically weaker sections will enable the well-off to corner benefits" which is available at https://indianexpress.com/article/opinion/columns/ews-general-category-quota-sc-st-supreme-court-5557300/ (both links last accessed on 04/14/2019).

[46]Also see the 01/10/2019 *The Economist* story "Quotas for all Almost all Indians will soon qualify for affirmative action in India," available at https://www.economist.com/asia/2019/01/10/almost-all-indians-will-soon-qualify-for-affirmative-action-in-india. Last accessed on 04/18/2019.

of India. The main arguments of the petitioners are reported in the *Supreme Court Observer* as follows:

(1) Reservations cannot be based solely on economic criteria, given the Supreme Court's judgment in *Indra Sawhney (1992)*.
(2) SCs/STs and OBCs cannot be excluded from economic reservations, as this would violate the fundamental right to equality.
(3) The Amendment introduces reservations that exceed the 50% ceiling-limit on reservations, established by *Indra Sawhney*.
(4) Imposing reservations on educational institutions that do not receive State aid violates the fundamental right to equality.

Our analysis and policy recommendation closely relate to this ongoing debate, where we present an important unintended consequence of the new Amendment: Since it decreases the fraction of the vertical reservation-ineligible population (i.e. the "new general category") to less than one tenth of the population in India, only the members of this population becomes eligible for horizontal adjustments under the Supreme Court-mandated choice rule, potentially amplifying the adverse consequences of shortcomings we presented in Section 4. Since EWS-women or EWS-disabled will no longer be eligible for horizontal adjustments for open-category positions, in many cases these positions will be awarded to lower score general-category women or disabled, which in turn will likely increase litigations of the sort we documented in Section 6. Bringing into light this subtle discord between the recent Amendment and the Supreme Court-mandated choice rule may provide valuable for the evaluation of both the Amendment and the choice rule.

References

Abdulkadiroğlu, Atila and Tayfun Sönmez, "School choice: A mechanism design approach," *American Economic Review*, June 2003, *93* (3), 729–747.

Andersson, Tommy, "Refugee Matching as a Market Design Application," 2017. Working Paper.

Aygün, Orhan and Bertan Turhan, "Dynamic Reserves in Matching Markets," *Unpublished paper*, 2016.

______ **and** ______, "Large-Scale Affirmative Action in School Choice: Admissions to IITs in India," *American Economic Review*, May 2017, *107* (5), 210–13.

______ **and Inácio Bó**, "College admission with multidimensional privileges: The Brazilian affirmative action case," *Unpublished paper, WZB Berlin Social Science Center.[869]*, 2016.

Balinski, Michel and Tayfun Sönmez, "A Tale of Two Mechanisms: Student Placement," *Journal of Economic Theory*, January 1999, *84* (1), 73–94.

Budish, Eric, "The Combinatorial Assignment Problem: Approximate Competitive Equilibrium from Equal Incomes," *Journal of Political Economy*, 2011, *119* (6), 1061–1103.

Chambers, Christopher P. and M. Bumin Yenmez, "Choice and Matching," *American Economic Journal: Microeconomics*, August 2017, *9* (3), 126–47.

Delacrétaz, David, Scott Duke Kominers, and Alexander Teytelboym, "Refugee Resettlement," 2016. Working Paper.

Dur, Umut Mert, Parag Pathak, and Tayfun Sönmez, "Explicit vs. Statistical Targeting in Affirmative Action: Theory and Evidence from Chicago's Exam Schools," *NBER Working Paper*, 2016, *22109.*

Dur, Umut, Scott Duke Kominers, Parag A. Pathak, and Tayfun Sönmez, "Reserve Design: Unintended Consequences and the Demise of Boston's Walk Zones," *Journal of Political Economy*, 2018, *126* (6), 2457–2479.

Echenique, Federico and M. Bumin Yenmez, "How to Control Controlled School Choice," *American Economic Review*, August 2015, *105* (8), 2679–2694.

Edelman, Benjamin, Michael Ostrovsky, and Michael Schwarz, "Internet Advertising and the Generalized Second-Price Auction: Selling Billions of Dollars Worth of Keywords," *American Economic Review*, March 2007, *97* (1), 242–259.

Ehlers, Lars, Isa E. Hafalir, M. Bumin Yenmez, and Muhammed A. Yildirim, "School choice with controlled choice constraints: Hard bounds versus soft bounds," *Journal of Economic Theory*, 2014, *153*, 648–683.

Fragiadakis, Daniel and Peter Troyan, "Improving matching under hard distributional constraints," *Theoretical Economics*, 2017, *12* (2), 863–908.

Hafalir, Isa E., M. Bumin Yenmez, and Muhammed A. Yildirim, "Effective affirmative action in school choice," *Theoretical Economics*, May 2013, *8* (2), 325–363.

Hafalir, Isa, Fuhito Kojima, and M. Bumin Yenmez, "Interdistrict School Choice: A Theory of Student Assignment," December 2018. Working paper.

Jones, Will and Alexander Teytelboym, "The Local Refugee Match: Aligning Refugees' Preferences with the Capacities and Priorities of Localities," *Journal of Refugee Studies*, 08 2017, *31* (2), 152–178.

Kamada, Yuichiro and Fuhito Kojima, "Efficient Matching under Distributional Constraints: Theory and Applications," *American Economic Review*, 2015, *105* (1), 67–99.

Kojima, Fuhito, "School choice: Impossibilities for affirmative action," *Games and Economic Behavior*, 2012, *75* (2), 685–693.

Kominers, Scott Duke and Tayfun Sönmez, "Matching with slot-specific priorities: Theory," *Theoretical Economics*, 2016, *11* (2), 683–710.

Milgrom, Paul, "Putting Auction Theory to Work: The Simultaneous Ascending Auction," *Journal of Political Economy*, April 2000, *108* (2), 245–272.

Neuborne, Burt, "The Supreme Court of India," *International Journal of Constitutional Law*, 2003, pp. 476–511.

Roth, Alvin E. and Elliott Peranson, "The Redesign of the Matching Market for American Physicians: Some Engineering Aspects of Economic Design," *American Economic Review*, September 1999, *89* (4), 748–780.

———, **Tayfun Sönmez, and M. Utku Ünver**, "Pairwise kidney exchange," *Journal of Economic Theory*, 2005, *125* (2), 151–188.

Roth, Alvin, Tayfun Sönmez, and Utku Ünver, "Kidney Exchange," *Quarterly Journal of Economics*, May 2004, *119* (2), 457–488.

Schummer, James and Azar Abizada, "Incentives in landing slot problems," *Journal of Economic Theory*, 2017, *170*, 29–55.

——— **and Rakesh V. Vohra**, "Assignment of Arrival Slots," *American Economic Journal: Microeconomics*, May 2013, *5* (2), 164–85.

Sönmez, Tayfun, "Bidding for Army Career Specialties: Improving the ROTC Branching Mechanism," *Journal of Political Economy*, 2013, *121* (1), 186–219.

——— **and Tobias B. Switzer**, "Matching With (Branch-of-Choice) Contracts at the United States Military Academy," *Econometrica*, 2013, *81* (2), 451–488.

Sönmez, Tayfun and Utku Ünver, "Course Bidding at Business Schools," *International Economic Review*, 2010, *51* (1), 99–123.

Varian, Hal R., "Position auctions," *International Journal of Industrial Organization*, 2007, *25* (6), 1163–1178.

Westkamp, Alexander, "An analysis of the German university admissions system," *Economic Theory*, 2013, *53* (3), 561–589.

Appendix A. The Relationship Between Incentive Compatibility and Elimination of Justified Envy

Definition 5. *Choice rule C satisfies* ***the irrelevance of rejected individuals*** *if for every $I \subseteq \mathcal{I}$ and $i \in R(I)$, $C(I) = C(I \setminus \{i\})$.*

Proposition 6. *Suppose that choice rule C is incentive compatible, respects inter se merit, and satisfies the irrelevance of rejected individuals. Then it eliminates justified envy.*

Definition 6. *Choice rule C satisfies* ***substitutability*** *if for every $I \subseteq \mathcal{I}$, $i \in C(I)$, and $j \neq i$, we have $i \in C(I \setminus \{j\})$.*

Proposition 7. *Suppose that choice rule C eliminates justified envy, satisfies substitutability and the irrelevance of rejected individuals. Then it is incentive compatible.*

In the next example, we provide a choice rule that eliminates justified envy and satisfies the irrelevance of rejected individuals but is not incentive compatible.

Example 6. Consider any set of individuals. If there exist at least two general-category individuals, choose two of them who have the highest merit scores. In this case, let the second highest merit score be s. Then we additionally choose all backward-class individuals who have merit scores greater than s. If the number of general-category individuals is one or zero, then no individual is chosen.

This choice rule eliminates justified envy because an individual with a higher score is never rejected while an individual with a lower score is accepted. It satisfies the irrelevance of rejected individuals trivially. It does not satisfy incentive compatibility because of the following situation. Suppose that there is only one general-category individual, then nobody is chosen. However, if any individual with a reserve-eligible category participates as if he was a general-category individual, then he would be chosen.

This observation is compatible with Proposition 7 because the choice rule does not satisfy substitutability. When two general-category individuals apply, they are both chosen. However, if only one of them applies, then this individual is not chosen. □

Appendix B. Proofs

In this appendix, we provide the omitted proofs.

Proof of Proposition 1. First, we show that C^{hor} is well-defined. Consider a set of individuals $I \subseteq \mathcal{I}$. In the construction of $C^{hor}(I)$, at every step, we consider subsets of I that satisfy the horizontal reservations for I. In the first part of the proof, we construct a subset of I that satisfies the horizontal reservations for I to show that there exists at least one such subset.

Consider all individuals in I who have at least one trait with a reserved position, say $\tilde{I}_1$. If $\tilde{I}_1$ is empty, then choose one individual in I. Otherwise, choose one individual from $\tilde{I}_1$ and decrease the number of reserved positions for the traits of this individual by one. Consider the set of remaining individuals in $\tilde{I}_1$ who have at least one trait with a reserved position, say $\tilde{I}_2$. If $\tilde{I}_2$ is empty, then choose one of the remaining individuals from I. Otherwise, if $\tilde{I}_2$ is not empty, then choose an individual from $\tilde{I}_2$. Continue this procedure so that the number of chosen individuals is $\min\{q, |I|\}$. We claim that the chosen subset, say I', satisfies the horizontal reservations for I. Suppose, for contradiction, that it does not. Then there exists a trait t such that the number of individuals with trait t in I' is less than r_t and that there is at least one individual in $I \setminus I'$ with trait t. In this case, $|I'| = q$ because $I \setminus I'$ is nonempty. Since the number of remaining reserved positions for trait t is positive, and an individual with this trait is not chosen at the last step, an individual with a trait that has a positive reserved position is chosen at every step. But this is a contradiction to the assumption that $\sum_{t \in \mathcal{T}} r_t \leq q$. Therefore, there exists at least one subset of I that satisfies the horizontal reservations for I.

Let $I' = \{i'_1, \ldots, i'_n\}$ be a subset of I that satisfies the horizontal reservations for I and $C^{hor}(I) = \{i_1, \ldots, i_m\}$. Suppose that $I' \neq C^{hor}(I)$. Re-order individuals in each set so that individuals with a lower index have higher merit scores than individuals with a higher index. We claim that $C^{hor}(I)$ dominates I'. Let k be the minimum index such that $i_k \neq i'_k$. By construction of $C^{hor}(I)$, i_k has a higher merit score than all individuals in $I' \setminus C^{hor}(I)$ because at Step k both $C^{hor}(I)$ and I' are considered and individual i_k is chosen by C^{hor}. Therefore, $C^{hor}(I)$ dominates I'. □

Proof of Proposition 2. Before we start the proof, we introduce some notation. For any set of individuals $I \subseteq \mathcal{I}$ and trait $t \in \mathcal{T}$, let $I_t \equiv \{i \in I | t \in \tau(i)\}$. In words, I_t is the set of individuals in I who have trait t. We use the following lemma in the proof.

Lemma 1. *$\bar{I} \subseteq I$ satisfies the horizontal reservations for I if, and only if, $|\bar{I}_t| \geq \min\{r_t, |I_t|\}$ for every trait $t \in \mathcal{T}$.*

Proof. First we show sufficiency. Let $\bar{I}$ be such that $|\bar{I}_t| \geq \min\{r_t, |I_t|\}$ for every trait t. Fix a trait t'. If $|\bar{I}_{t'}| < r_{t'}$, then $|\bar{I}_{t'}| \geq |I_{t'}|$. Since $\bar{I} \subseteq I$, this implies $\bar{I}_{t'} = I_{t'}$. Therefore, there exists no individual in $I \setminus \bar{I}$ who has trait t'. Therefore, $\bar{I}$ satisfies the horizontal reservations for I.

For necessity, let $\bar{I} \subseteq I$ satisfy the horizontal reservations for I. Then, for every trait t, either $|\bar{I}_t| \geq r_t$ or $\bar{I}_t = I_t$. This implies $|\bar{I}_t| \geq \min\{r_t, |I_t|\}$ for every trait $t \in \mathcal{T}$. ■

Consider a set of individuals I. We show that $C^{hor}(I) = C^{mg}(I)$. First, for every trait t, the number of trait-t individuals in $C^{mg}(I)$ is at least $\min\{r_t, |I_t|\}$ because in the first step $\min\{r_t, |I_t|\}$ trait-t individuals are chosen. Therefore, by Lemma 1, $C^{mg}(I)$ satisfies the horizontal reservations for I. Let I' be the set of individuals chosen out of I by C^{mg} in Step 1. $C^{hor}(I)$ must include all the individuals in I' because by Lemma 1 the number of trait-t individuals in $C^{hor}(I)$ is at least $\min\{r_t, |I_t|\}$. Furthermore, by the construction of C^{hor}, whenever a trait-t individual is chosen, it always selects the trait-t individual with the highest merit score from the available set, so $C^{hor}(I) \supseteq I'$.

Now, if $C^{mg}(I) \setminus I' \neq C^{hor}(I) \setminus I'$, then $C^{mg}(I) \setminus I'$ would dominate $C^{hor}(I) \setminus I'$ by the construction of C^{mg} because it selects individuals with the highest merit score in Step 2. Therefore, $C^{mg}(I)$ would dominate $C^{hor}(I)$ because adding or subtracting a set of individuals preserves the domination relationship. But this cannot hold because C^{hor} is merit maximal and so $C^{hor}(I)$ dominates any subset of I different from $C^{hor}(I)$ that satisfies the horizontal reservations for I. Therefore, $C^{mg}(I) \setminus I' = C^{hor}(I) \setminus I'$, and thus $C^{mg}(I) = C^{hor}(I)$. □

Proof of Proposition 3. Denote the union of the set of individuals with highest r^o merit scores and the set of all general-category individuals by I_1. In Step 1 of C^{SCI}_{1h},

$C^{hor}(I_1|r^o,(r^o_t)_{t\in\mathcal{T}})$ is chosen for the open-category positions. We first show that the set of individuals chosen for the open-category positions by C^{SCI} is the same set.

When the open-category positions are allocated according to C^{SCI} in Steps $1(i)$ and $1(ii)$, only individuals in I_1 are considered. Furthermore, the chosen set, call it I', satisfies the open-category horizontal reservations $(r^o_t)_{t\in\mathcal{T}}$ for I_1 because for each trait t either the number of chosen trait-t individuals is at least r^o_t or all trait-t individuals in I_1 are chosen. Likewise, $C^{hor}(I_1|r^o,(r^o_t)_{t\in\mathcal{T}})$ also satisfies the open-category horizontal reservations $(r^o_t)_{t\in\mathcal{T}}$ for I_1 by Proposition 1.

By Lemma 1, for each trait t, the number of trait-t individuals in I' and $C^{hor}(I_1|r^o,(r^o_t)_{t\in\mathcal{T}})$ are at least $\min\{r^o_t,|\{i\in I_1|\tau(i)=t\}|\}$. In both choice rules, a trait-t individual with a low merit score is never chosen before a trait-t individual with a higher merit score, so for every trait t, trait-t individuals with the highest $\min\{r^o_t,|\{i\in I_1|\tau(i)=t\}|\}$ merit scores in I' and $C^{hor}(I_1|(r^o_t)_{t\in\mathcal{T}})$ are the same. Furthermore, for the rest of the individuals chosen in Step 1, both rules choose individuals with the highest merit scores remaining in I_1, so they must choose the same set of individuals.

Next, we show that the set of individuals chosen for each reserve-eligible category $c\in\mathcal{C}$ is the same in C^{SCI}_{1h} and C^{SCI}. First note that the set of category-c individuals considered for positions at the second step are the same in both choice rules. The rest of the proof is analogous to the discussion above, as the same set of individuals are considered and the same procedures are applied at this step. □

Proof of Proposition 4. To show elimination of justified envy of C^{hor}_{2s}, consider a set of individuals I and two individuals $i,i'\in I$ with $\rho(i)\subseteq\rho(i')$, $\tau(i)\subseteq\tau(i')$ and $\sigma(i)<\sigma(i')$. At any step when i is considered by C^{hor}_{2s}, i' is also considered. Furthermore, by the construction of C^{hor}, which is used at every step of C^{hor}_{2s}, an individual with a lower merit score and set of traits τ is never chosen before another individual with a higher merit score and set of traits τ' where $\tau'\supseteq\tau$. Therefore, C^{hor}_{2s} eliminates justified envy.

To show incentive compatibility of C^{hor}_{2s}, consider a set of individuals I and an individual $i\in I$ such that $i\notin C^{hor}_{2s}(I)$. Fix every other individual's category and set of traits. First note that C^{hor} does not use the categories of individuals, so modifying the category of i from a reserve-eligible category to general can only hurt i, as he will only be considered at the first step. Furthermore, declaring a set of traits $\tau\subsetneq\tau(i)$ instead of $\tau(i)$ can only make this individual worse off, because if he is considered with set of traits τ to satisfy some constraints, then he will also be considered with set of traits $\tau(i)$ to satisfy the same constraints. Therefore, C^{hor}_{2s} is incentive compatible.

To show elimination of justified envy of C^{hor}_{ite}, consider a set of individuals I and two individuals $i,i'\in I$ with with $\rho(i)\subseteq\rho(i')$, $\tau(i)\subseteq\tau(i')$, and $\sigma(i)<\sigma(i')$. At every Step k, where $k\geq 3$, when i is considered by C^{hor}_{2s} i' is also considered. Furthermore, by the construction of C^{hor}, which is used at every step of C^{hor}_{2s}, an individual with a lower merit score and set of traits τ is never chosen before another individual with a higher merit score and set of traits τ' where $\tau' \supseteq \tau$. Since C^{hor}_{ite} terminates at Step 3 or later, C^{hor}_{ite} eliminates justified envy.

To show incentive compatibility of C^{hor}_{ite}, consider a set of individuals I and an individual $i \in I$ such that $i \notin C^{hor}_{ite}(I)$. Fix every other individual's category and set of traits. First note that C^{hor} does not use the categories of individuals and unassigned individuals with reserve eligible categories is considered for the open positions at Step 3 and later, so modifying the category of i from a reserve-eligible category to general can only hurt i, as he will not be considered for the positions reserved for his category. Furthermore, declaring a set of traits $\tau \subseteq \tau(i)$ instead of $\tau(i)$ can only make this individual worse off, because if he is considered with set of traits τ to satisfy some constraints, then he will also be considered with set of traits $\tau(i)$ to satisfy the same constraints. Therefore, C^{hor}_{2s} is incentive compatible. □

Proof of Proposition 5. When each individual has at most one trait, by Proposition 1, C^{hor} is the same as C^{mg}, which satisfies substitutability (Echenique and Yenmez, 2015, Theorem 2). In addition, C^{hor} also satisfies the irrelevance of rejected individuals. Therefore, C^{hor} satisfies path independence, i.e., for every $I, I' \subseteq \mathcal{I}$, $C^{hor}(I \cup I') = C^{hor}(C^{hor}(I) \cup C^{hor}(I'))$.[47]

Let $I \subseteq \mathcal{I}$ be a set of applicants and $I_1 \subseteq I$ be the set of applicants considered at Step 0 of C^{hor}_{ite} and C^{SCI}.

By path independence of C^{hor}, $C^{hor}_{ite}(I) \cap I^G = C^{hor}(I_2) \cap I^G$ where I_2 is the set of individuals considered for the open-category positions at any step of C^{hor}_{ite}. Since $I_2 \supseteq I_1$ and $C^{hor}(I_2) \cap I^G \subseteq I_1$, by substitutability of C^{hor}, $C^{hor}(I_1) \cap I^G \supseteq C^{hor}(I_2) \cap I^G$, which is equivalent to $C^{SCI}(I) \cap I^G \supseteq C^{hor}_{ite}(I) \cap I^G$. Similarly, since $I \supseteq I_2$ and $C^{hor}(I) \cap I^G \subseteq I_2$, by substitutability of C^{hor}, $C^{hor}(I_2) \cap I^G \supseteq C^{hor}(I) \cap I^G$, which is equivalent to $C^{hor}_{ite}(I) \cap I^G \supseteq C^{hor}_{2s}(I) \cap I^G$.

Since the number of general-category individuals who get a position under C^{SCI} is weakly more than the number of general-category individuals who get a position under C^{hor}_{ite} and C^{hor} does not reject an individual unless the capacity is filled, the number of individuals with a reserve-eligible category who receive a position under C^{SCI} is weakly less than the number of individuals with a reserve-eligible category who receive a position under C^{hor}_{ite}. Similarly, the number of individuals with a reserve-eligible category who receive a position under C^{hor}_{ite} is weakly less than the number of individuals with a reserve-eligible category who receive a position under C^{hor}_{2s}. □

[47]See, for example, Chambers and Yenmez (2017) for path independence and its application in a matching context.

Proof of Theorem 1. *Proof of Part a*: When I is the set of applicants, let $I_1 \subseteq I$ be the set of individuals who are considered at Step 0 of C^{SCI} (or C^{hor}_{ite}), $I^o \subseteq I$ be the set individuals who are allocated open-category positions by C^{SCI}, and $I^u \subseteq I$ be the set of individuals who are not allocated any positions by C^{SCI}.

To show that $C^{SCI}(I) = C^{hor}_{ite}(I)$, we need to prove that C^{hor}_{ite} terminates at Step 3 when I is the set of applicants. Since Steps 0, 1, and 2 are the same in C^{SCI} and C^{hor}_{ite}, we need $C^{hor}(I^o \cup I^u | r^o, (r^o_t)_{t\in\mathcal{T}}) = I^o$ to show the result. We prove a more general result that $C^{hor}(I_1 \cup I^u | r^o, (r^o_t)_{t\in\mathcal{T}}) = I^o$, which implies $C^{hor}(I^o \cup I^u | r^o, (r^o_t)_{t\in\mathcal{T}}) = I^o$ because $I_1 \supseteq I^o$ and the fact that C^{hor} satisfies the irrelevance of rejected individuals. For the rest of the proof, we use C^{hor} with parameters $(r^o, (r^o_t)_{t\in\mathcal{T}})$, and, to simplify the notation, we omit them.

Since each individual has at most one trait, by Proposition 1, C^{hor} is the same as C^{mg}, which satisfies substitutability (Echenique and Yenmez, 2015, Theorem 2). By substitutability of C^{hor}, any individual rejected when I_1 is the set of applicants is also rejected when $I_1 \cup I^u$ is the set of applicants. Therefore, we only consider individuals in I^u who have a reserve-eligible category.

For any individual $j \in I^u$ with a reserve-eligible category who does not have a trait, there are at least r^o number of individuals in I_1 who have higher merit scores than j. Therefore, individual j cannot be chosen by C^{hor} when $I_1 \cup I^u$ is the set of applicants.

For any individual $j \in I^u$ with a reserve-eligible category and a trait, say t, there are at least $\min\{r^o_t, |i \in I : \tau(i) = t|\}$ number of individuals with trait t in I^o who have strictly higher merit scores than j. This holds because, by construction of I_1, every individual with a reserve eligible category in I^o have a higher merit score than j and every general-category individual with trait t in I^o have higher merit scores than j since $C^{SCI}(I)$ eliminates justified envy.

Therefore, an individual in I^u with a reserve-eligible category cannot be accepted because of horizontal reservations for the open seats. Furthermore, since $I^u \cap I_1 = \emptyset$, there are at least r^o number of individuals in I_1 who have higher merit scores than every individual in I^u. Hence, no individual in I^u can be chosen by C^{hor} when $I_1 \cup I^u$ is the set of applicants. Thus, $C^{hor}(I_1 \cup I^u) = C^{hor}(I_1)$ by the irrelevance of rejected individuals, which implies that $C^{hor}(I_1 \cup I^u) = I^o$ because $C^{hor}(I_1) = I^o$.

Proof of Part b: Suppose that $C^{SCI}(I)$ does not eliminate justified envy, $\left|C^{SCI}(I)\right| = q$, and there is one trait t. Then the instances of justified envy involve a trait-t individual with a reserve-eligible category and a general-category individual with the same trait.

Let I^G_t be the set of general-category individuals with trait t who are given an open-category position and I^R_t be the set of individuals with a reserve-eligible category and trait t who are unassigned such that every individual in I_t^G is justifiably envied by someone in I_t^R and every individual in I_t^R justifiably envies someone in I_t^G. Let k_t be the maximum integer such that the individual in I_t^G with the k-th lowest merit score is lower than the individual in I_t^R with the k-th highest merit score. Since $C^{SCI}(I)$ does not eliminate justified envy, $k_t > 0$. By construction, $\left|C^{SCI}(I) \setminus C_{ite}^{hor}(I)\right| = k_t$ because $C_{ite}^{hor}(I)$ replaces k_t individuals in I_t^G with the lowest merit scores, denote this set by A_t, with k_t individuals in I_t^R with the highest merit scores, denote this set by B_t.

We consider two cases. If $A_t \cap C(I) = \emptyset$, then $C^{SCI}(I) \setminus C(I) \supseteq A_t$. Therefore, $\left|C^{SCI}(I) \setminus C(I)\right| \geq |A_t| = k_t = \left|C^{SCI}(I) \setminus C_{ite}^{hor}(I)\right|$. Otherwise, if $A_t \cap C(I) \neq \emptyset$, then $C(I) \setminus C^{SCI}(I) \supseteq B_t$ because $C(I)$ eliminates justified envy and every individual in B_t has a reserve-eligible category, trait t and a higher merit score than all individuals in A_t who have general category and trait t. Since $\left|C^{SCI}(I)\right| = q$, $|C(I)| \leq q$, and $\left|C(I) \setminus C^{SCI}(I)\right| \geq |B_t| = k_t$, we get $\left|C^{SCI}(I) \setminus C(I)\right| \geq k_t = \left|C^{SCI}(I) \setminus C_{ite}^{hor}(I)\right|$.

□

Proof of Proposition 6. Suppose, for contradiction, that C is incentive compatible, respects inter se merit, and satisfies the irrelevance of rejected individuals but it does not eliminate justified envy. Then, there exist $I \subseteq \mathcal{I}$, $i, j \in I$ with $\alpha(i) \subseteq \alpha(j)$, and $\sigma(i) < \sigma(j)$ such that $i \in C(I)$ and $j \in R(I)$. By incentive compatibility, if j witholds some of her reserve-eligible attributes and treated as an individual with the set of attibutes $\alpha(i)$, then she will not be chosen. Call this hypothetical individual $\tilde{j}$, so $\alpha(\tilde{j}) = \alpha(i)$ and $\sigma(\tilde{j}) = \sigma(j) > \sigma(i)$. Thus, $\tilde{j} \notin C\big((I \setminus \{j\}) \cup \{\tilde{j}\}\big)$.

By the irrelevance of rejected individuals, $C\big((I \setminus \{j\}) \cup \{\tilde{j}\}\big) = C(I \setminus \{j\})$ and $C(I) = C(I \setminus \{j\})$. Therefore,

$$C\big((I \setminus \{j\}) \cup \{\tilde{j}\}\big) = C(I),$$

which implies that $i \in C\big((I \setminus \{j\}) \cup \{\tilde{j}\}\big)$ because $i \in C(I)$. This contradicts the assumption that C respects inter se merit because $\alpha(\tilde{j}) = \alpha(i)$ and $\sigma(\tilde{j}) > \sigma(i)$, but $\tilde{j}$ is rejected while i is chosen from $(I \setminus \{j\}) \cup \{\tilde{j}\}$. □

Proof of Proposition 7. Suppose, for contradiction, that C eliminates justified envy, satisfies substitutability and the irrelevance of rejected individuals but is not incentive compatible. Then there exist a set of individuals I and an individual $i \in I$ such that i is chosen from I when she withholds some of her reserve-eligible privileges and $i \notin C(I)$. Let $\tilde{i}$ be the hypothetical individual when i holds some of her privileges, so $\alpha(\tilde{i}) \subseteq \alpha(i)$ and $\sigma(\tilde{i}) = \sigma(i)$. Therefore, $\tilde{i} \in C\big((I \cup \{\tilde{i}\}) \setminus \{i\}\big)$.

Substitutability and $i \notin C(I)$ imply that $i \notin C(I \cup \{\tilde{i}\})$. By the irrelevance of rejected individuals, $C(I \cup \{\tilde{i}\}) = C\big((I \cup \{\tilde{i}\}) \setminus \{i\}\big)$. This implies $\tilde{i} \in C(I \cup \{\tilde{i}\})$, because $\tilde{i} \in$

$C\big((I \cup \{\tilde{i}\}) \setminus \{i\}\big)$. This is a contradiction to elimination of justified envy because $i \in R(I \cup \{\tilde{i}\})$, $\tilde{i} \in C(I \cup \{\tilde{i}\})$, $\alpha(\tilde{i}) \subseteq \alpha(i)$ and $\sigma(\tilde{i}) = \sigma(i)$. □

Appendix C. Potential Over Adjustment for Horizontal Traits Under C^{SCI}

Example 7. Consider a set of candidates with four men m_1, m_2, m_3, m_4 and two women w_1, w_2. Candidates w_2 and m_4 are disabled. There are three open positions. There is one horizontal reservation for the female candidates and one horizontal reservation for the disabled candidates. Starting with the candidate with the highest merit score, the candidates are ranked according to their merit scores as follows:

$$\sigma(m_1) > \sigma(m_2) > \sigma(m_3) > \sigma(w_1) > \sigma(m_4) > \sigma(w_2).$$

The SCI-VHR choice rule works as follows: Initially the three highest merit score candidates m_1, m_2, m_3, are selected for the three open positions. Since all of these candidates are men and none of them is disabled, neither the minimum guarantee for female candidates nor the minimum guarantee for disabled candidates is satisfied. While the adjustment process to accommodate the horizontal reservations is clearly indicated in *Rajesh Kumar Daria (2007)* for a single trait of horizontal reservations, this reference court case fails to specify how to proceed with the adjustment process when there are multiple traits of horizontal reservations. While it is not always the case, in this example the processing order of the horizontal reservation traits is immaterial. Thus, suppose that the adjustment process starts with the horizontal reservation constraint for female candidates. In that case, the highest merit score female candidate w_1 has to be chosen at the expense of the male candidate m_3 by the mechanics of the adjustment process given in *Rajesh Kumar Daria (2007)* described above. Next, the disabled horizontal reservation constraint is accommodated by including the highest score disabled candidate m_4 in the choice set, at the expense of a second displaced candidate m_2. At this point, both horizontal reservation constraints are satisfied, and the outcome of the SCI-VHR choice rule is finalized as

$$\{m_1, w_1, m_4\}.$$

Observe that the same outcome is obtained if the disabled horizontal reservation is accommodated first and the female horizontal reservation is accommodated next.

Therefore, through the adjustment phase, two higher merit-score candidates m_2 and m_3 are removed from the original merit-based choice set. We argue that the removal of the candidate m_2 is unjustified since both horizontal reservation constraints could have been accommodated with only one adjustment, namely by including the disabled female candidate w_2 at the expense of the candidate m_3. When the SCI-VHR choice rule was originally introduced, the judges of the Supreme Court in *Anil Kumar Gupta (1995)* indicated that, for the purpose of accommodating the horizontal reservations "the requisite number of special reservation candidates shall have to be taken and adjusted/accommodated against their respective social reservation categories by deleting the corresponding number of candidates therefrom." Since both of the special horizontal reservations can be satisfied with the inclusion of the disabled female candidate w_2, we argue that the requisite number is only one. The outcome that has to be selected with only one adjustment is

$$\{m_1, m_2, w_2\}.$$

But this outcome cannot be achieved by accommodating the horizontal reservation types one at a time. Instead, a forward-looking approach is needed for the adjustment phase. □

Appendix D. Case Study: Ashish Sharma & Ors. vs. State Of Chhattisgarh & Ors. on August 18th, 2003

In this Chhattisgarh High Court case, the petitioners challenge the implementation of horizontal reservations for women at a Chhattisgarh Medical School. There are 42 open seats, of which 13 are horizontally reserved for women, one is horizontally reserved for soldiers, and one is horizontally reserved for freedom fighters. In order to allocate the 42 open seats, the respondents followed a procedure that is mechanically different from the procedure for SCI-VHR choice rule C^{SCI}: They first allocated 13 seats to the highest merit score women, next allocated 27 seats to the remaining highest score candidates bringing the total to 40, and since horizontal reserves for soldiers and freedom fighters were not satisfied by this point, they assigned one seat each to the remaining candidates with the highest merit who has one of these two traits.[48] In addition to the 13 seats allocated to women in the first step, an additional 12 seats were also allocated to women among the 27 seats allocated in the second step, for a total of 25 seats. Observe that by Propositions 2 and 3, the procedure followed by the respondents gives the same outcome as the choice rule C^{SCI}. However, failing to observe this equivalence, the male petitioners challenged the procedure used by the respondents. This equivalence was not explained clearly by counsel, which in turn resulted the judges of the high court siding with the petitioners, requiring them to repeat the allocation process using the SCI-VHR choice rule.

[48]The exact treatment of the one unit of horizontal reserve for soldiers and one unit of horizontal reserve for freedom fighters is not described in the case, and this last step is our interpretation from the description in the case.