Student Placement in Turkey

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Based on mostly:

Balinski & Sönmez (*JET* 1999)

Two-Sided Matching Markets & Student Placement

- Two-Sided Matching Markets (Gale & Shapley 1962)
  - Models (many-to-one) two-sided matching markets

    Example: Hospital-intern matching in the U.S.
  - Both schools and students are (potentially strategic) agents
- Student Placement (Balinski & Sönmez 1999)
  - Models centralized school admissions.

    Example: University admissions in Turkey.
  - Students are (potentially) strategic agents
  - School seats are goods to be consumed
  - Priority at schools determined by exam scores
  - Under an adequate "fairness" axiom, model isomorphic to stable two-sided matching markets.

Student Placement: The Model

A student placement problem consists of

$$\begin{array}{lcll} I & = & \{i_1, \ldots, i_n\} & \text{a set of students} \\ C & = & \{c_1, \ldots, c_m\} & \text{a set of colleges} \\ R & = & (R_{i_1}, \ldots, R_{i_n}) & \text{a list of student preferences} \\ q & = & (q_1, \ldots, q_m) & \text{a vector of college capacities} \\ T & = & \{t_1, \ldots, t_k\} & \text{a set of skill categories} \\ f & = & (f^{i_1}, \ldots, f^{i_n}) & \text{a list of test scores} \\ t & : & C \longrightarrow T & \text{a function from } C \text{ to } T \end{array}$$

Student Placement: The Model

Here

- q_c is the capacity of college c,
- R_i is the preference of student i over colleges and the no college option,
- $f^i = (f^i_{t_1}, \ldots, f^i_{t_k})$ is a vector which gives the test score of student i in each category, and
- t is a function which maps each college to a category.

Matching

- Definition: A matching is a function $\mu : I \longrightarrow C \cup \{\emptyset\}$ such that no college is assigned to more students than its capacity.

  $\mu(i) = \emptyset$: Student i is unmatched.

- Definition: A student placement mechanism (or a mechanism in short) is a function that assigns a matching for each problem.

Fairness

- Definition: A matching μ is fair if no student i loses a seat to another student j who has lower score in the category of school $\mu(j)$.
- Critical in the context of Turkish college admissions.
- Definition: A mechanism is fair if it always selects a matching that is fair.

Simple Case: One Skill Category

- Practical Application: Assignment of students to high schools in Turkey via SBS exam.
- Definition: Given a priority ranking, the induced simple serial dictatorship assigns the first student his top choice, the next student his top choice among remaining seats, etc.
- Proposition: If there is only one category (and hence only one priority ranking) then there is only one mechanism that is fair and Pareto efficient: The simple serial dictatorship induced by this ranking.

Simple Case: One Skill Category

- Practical Application: Assignment of students to high schools in Turkey via SBS exam.
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- Proposition: If there is only one category (and hence only one priority ranking) then there is only one mechanism that is fair and Pareto efficient: The simple serial dictatorship induced by this ranking.
- Bottomline: Choice of assignment mechanism is straightforward when students are priority ranked in a single list (as in the case of high school admissions).

Other Criteria

- Definition: A matching is individually rational if no student prefers the no college option to his assignment.
- Definition: A matching is non-wasteful if no student prefers a college with one or more empty slots to his assignment.

Turkish Mechanism: Multi-Category Serial Dictatorship

Step 1:

- For each category t: Consider the ranking induced by the test scores in this category and assign the relevant seats to students with the induced simple serial dictatorship.
- Assign the no college option to any unmatched student.
- This, in general, may not lead to a feasible student placement: Some students may be assigned slots at multiple colleges. To correct this, student preferences are truncated.
- For each student i construct R_i^1 from R_i as follows:
  - If the student is not assigned more than one college then $R_i^1 = R_i$.
  - If the student is assigned more than one college then obtain R_i^1 by moving the no college option $\emptyset$ right after the best of these assignments and otherwise keeping the ranking of the colleges the same.

Let R^1 be the list of adjusted preferences.

Turkish Mechanism: Multi-Category Serial Dictatorship

Step k: Construct R^k from R^{k-1} as it is described in Step 1.

Termination of the algorithm:

- The procedure terminates at the step in which no student is assigned more than one college.
- The Turkish mechanism (denoted by φ^{Turkish}) selects the resulting matching.

Example:

$$\begin{aligned}
\text{Students} &= \{\mathrm{Alp}, \mathrm{Banu}, \mathrm{Can}, \mathrm{Derin}, \mathrm{Elif}\} \\
\text{Colleges} &= \{c_1, c_2, c_3\} \\
\text{College capacities} &= (2, 1, 1) \\
\text{Skill Categories} &= \{\mathrm{MF}, \mathrm{TM}\} \\
t(c_1) &= \mathrm{MF} \\
t(c_2) = t(c_3) &= \mathrm{TM}
\end{aligned}$$

Student preferences and exam scores are as follows:

$$\begin{aligned}
R_{\mathrm{A}} &: \; c_2 - c_1 - \emptyset & f^{\mathrm{A}} &= (450, 450) \\
R_{\mathrm{B}} &: \; c_1 - c_2 - c_3 - \emptyset & f^{\mathrm{B}} &= (400, 300) \\
R_{\mathrm{C}} &: \; c_1 - c_3 - c_2 - \emptyset & f^{\mathrm{C}} &= (350, 350) \\
R_{\mathrm{D}} &: \; c_1 - c_2 - \emptyset & f^{\mathrm{D}} &= (300, 400) \\
R_{\mathrm{E}} &: \; c_2 - c_3 - c_1 - \emptyset & f^{\mathrm{E}} &= (250, 250)
\end{aligned}$$

Note that these scores induce the following rankings in each category:

$$\mathrm{MF} : \mathrm{A\ B\ C\ D\ E} \qquad \mathrm{TM} : \mathrm{A\ D\ C\ B\ E}$$

Step 1:

$$\text{MF}: \begin{array}{ccccc} \text{A} & \text{B} & \text{C} & \text{D} & \text{E} \\ c_1 & c_1 & & & \end{array} \qquad \text{TM}: \begin{array}{ccccc} \text{A} & \text{D} & \text{C} & \text{B} & \text{E} \\ c_2 & - & c_3 & & \end{array}$$

Step 1 yields the following tentative student placement:

$$\nu^1 = \begin{pmatrix} \text{Alp} & \text{Banu} & \text{Can} & \text{Derin} & \text{Elif} \\ c_1, c_2 & c_1 & c_3 & \emptyset & \emptyset \end{pmatrix}$$

Having assigned at least one slot, preferences of students $\text{Alp}, \text{Banu}, \text{Can}$ are truncated:

$$\begin{array}{ll} R^1_{\text{A}}: & c_2 - \emptyset \\ R^1_{\text{B}}: & c_1 - \emptyset \\ R^1_{\text{C}}: & c_1 - c_3 - \emptyset \end{array}$$

For other students: $R^1_{\text{D}} = R_{\text{D}}$, and $R^1_{\text{E}} = R_{\text{E}}$.

Step 2: In Step 2 we first find the serial dictatorship outcomes for R^1.

$$\mathrm{MF}: \begin{array}{ccccc} \mathrm{A} & \mathrm{B} & \mathrm{C} & \mathrm{D} & \mathrm{E} \\ - & c_1 & c_1 & & \end{array} \qquad \mathrm{TM}: \begin{array}{ccccc} \mathrm{A} & \mathrm{D} & \mathrm{C} & \mathrm{B} & \mathrm{E} \\ c_2 & - & c_3 & & \end{array}$$

Step 2 yields the following tentative student placement:

$$\nu^2 = \begin{pmatrix} \mathrm{Alp} & \mathrm{Banu} & \mathrm{Can} & \mathrm{Derin} & \mathrm{Elif} \\ c_2 & c_1 & c_1, c_3 & \emptyset & \emptyset \end{pmatrix}$$

Having assigned two slots, preferences of student Can is truncated:

$$R^2_{\mathrm{C}} : c_1 - \emptyset$$

For other students: $R^2_{\mathrm{A}} = R^1_{\mathrm{A}}$, $R^2_{\mathrm{B}} = R^1_{\mathrm{B}}$, $R^2_{\mathrm{D}} = R^1_{\mathrm{D}}$, and $R^2_{\mathrm{E}} = R^1_{\mathrm{E}}$.

Step 3: In Step 3 we first find the serial dictatorship outcomes for R^2.

$$\text{MF}: \begin{array}{ccccc} \text{A} & \text{B} & \text{C} & \text{D} & \text{E} \\ - & c_1 & c_1 & & \end{array} \qquad \text{TM}: \begin{array}{ccccc} \text{A} & \text{D} & \text{C} & \text{B} & \text{E} \\ c_2 & - & - & - & c_3 \end{array}$$

Step 3 yields the following tentative student placement (which is also a matching):

$$\nu^3 = \begin{pmatrix} \text{Alp} & \text{Banu} & \text{Can} & \text{Derin} & \text{Elif} \\ c_2 & c_1 & c_1 & \emptyset & c_3 \end{pmatrix}$$

Since no student is assigned more than one slot in ν^3, the algorithm terminates resulting in:

$$\varphi^{\text{Turkish}}(R, f, q) = \nu^3$$

Competing Mechanism: Student Proposing Deferred Acceptance (SPDA)

Step 1: Each student proposes to her first choice. Each school tentatively assigns its seats to its proposers one at a time following their priority order. Any remaining proposers are rejected.

In general, at

Step k: Each student who was rejected in the previous step proposes to her next choice. Each school considers the students it has been holding together with its new proposers and tentatively assigns its seats to these students one at a time following their priority order. Any remaining proposers are rejected.

A Disturbing Equivalence

- Gale & Shapley (1962) also introduced a college proposing version of the Deferred Acceptance algorithm.

  Resulting mechanism: College Proposing Deferred Acceptance (CPDA)
- Theorem (Gale & Shapley 1962): Of all individually rational, non-wasteful and fair allocations, CPDA assigns students the worst possible assignment!
- Theorem: Turkish Mechanism = CPDA

  What it means for Turkey: The above two results immediately show that Turkish mechanism results in unnecessary welfare loss by assigning students to their lower ranked choices than its possible!

Pareto Efficiency

Example: $I = \{\mathrm{Alp}, \mathrm{Banu}\}, \quad C = \{c_1, c_2\}, \quad q = (1,1),$
$T = \{\mathrm{MF}, \mathrm{TM}\}, \quad t(c_1) = \mathrm{MF},\ t(c_2) = \mathrm{TM}$

$$R_{\mathrm{A}} : c_1 - c_2 - \emptyset \qquad f^{\mathrm{A}} = (300, 400)$$
$$R_{\mathrm{B}} : c_2 - c_1 - \emptyset \qquad f^{\mathrm{B}} = (400, 300)$$

The algorithm terminates in one step resulting in the following Pareto inefficient matching:

$$\varphi^{\mathrm{Turkish}}(R, f, q) = \begin{pmatrix} \mathrm{Alp} & \mathrm{Banu} \\ c_2 & c_1 \end{pmatrix}$$

Bottom line: The Turkish mechanism assigns both students their second choices when they could have been assigned their first choices!

Pareto Efficiency

- Theorem (Gale & Shapley 1962): SPDA Pareto dominates any other fair mechanism (including the Turkish mechanism).
- Implication for Turkey: There is unnecessary efficiency loss under the Turkish mechanism.

  Adoption of SPDA will assure that each student is assigned to the best department that is possible under a fair allocation.

Strategy-Proofness

Example continued: Recall that

$$\varphi^{\text{Turkish}}(R, f, q) = \begin{pmatrix} \text{Alp} & \text{Banu} \\ c_2 & c_1 \end{pmatrix}$$

where both students are assigned their second choices.

Now suppose Alp announces a fake preference relation $\tilde{R}_{\text{A}}$ where only his first choice c_1 is acceptable. In this case

$$\varphi^{\text{Turkish}}(\tilde{R}_{\text{A}}, R_{\text{B}}, f, q) = \begin{pmatrix} \text{Alp} & \text{Banu} \\ c_1 & c_2 \end{pmatrix}$$

where Alp receives his first choice!

Bottom line: Alp succesfully manipulates the Turkish mechanism.

Strategy-Proofness

- Definition: A mechanism is strategy-proof if truthtelling is always an optimal strategy in its associated preference revelation game.
- Theorem (Dubins & Freedman 1981, Roth 1982): SPDA is *strategy-proof*.
- Theorem (Alcalde & Barberà 1994): SPDA is the only mechanism that is *individually rational, non-wasteful, fair* and *strategy-proof*.
- Implication for Turkey: Students can game they system by misrepresenting their preferences under the Turkish mechanism.

  Adoption of SPDA will assure that truthful ranking of schools is always optimal.

Respecting Improvements

Example further continued: Recall that

$$\varphi^{\text{Turkish}}(R, f, q) = \begin{pmatrix} \text{Alp} & \text{Banu} \\ c_2 & c_1 \end{pmatrix}$$

where both students are assigned their second choices.

Now suppose Alp scores worse in both tests and his new test scores are $\tilde{f}^{\text{A}} = (250, 250)$. In this case

$$\varphi^{\text{Turkish}}(R, \tilde{f}^{\text{A}}, f^{\text{B}}, q) = \begin{pmatrix} \text{Alp} & \text{Banu} \\ c_1 & c_2 \end{pmatrix}$$

where Alp receives his first choice!

Bottom line: Alp is revarded by getting his top choice as a result of inferior test scores!

Respecting Improvements

- Definition: A mechanism respects improvements if a student never receives a worse assignment as a result of an increase in one or more of his test scores.
- Theorem: SPDA respects improvements.
- Theorem: SPDA is the only mechanism that is *individually rational*, *non-wasteful*, *fair* and *respects improvements*.
- Implication for Turkey: Students can receive worse assignments due to an increase in their scores under the Turkish mechanism.

  Adoption of SPDA will assure that students can only benefit from higher scores.

Summary: Turkish Mechanism vs. SPDA

- The Turkish mechanism has three key deficiencies:
    1. It assigns students to potentially lower ranked schools then it might be possible by other fair mechanisms.
    2. Students can potentially game the system receiving better assignments by misrepresenting their preferences.
    3. Increasing their scores can occasionally harm students.
- Not only adopting SPDA resolves all these failures, it is the only fair mechanism to do so!

Summary: Turkish Mechanism vs. SPDA

- The Turkish mechanism has three key deficiencies:
  1. It assigns students to potentially lower ranked schools then it might be possible by other fair mechanisms.
  2. Students can potentially game the system receiving better assignments by misrepresenting their preferences.
  3. Increasing their scores can occasionally harm students.
- Not only adopting SPDA resolves all these failures, it is the only fair mechanism to do so!

- Bottomline: In an environment where fairness cannot be sacrificed (eg. when priorities obtained through exams as in Turkey), SPDA is the unambiguous winner!

Subsequent Developments

- 2003: Abdulkadiroğlu & Sönmez (2003) has shown that student placement mechanisms used by several major U.S. school districts suffer from similar deficiencies as the Turkish mechanism.

  They advocated adoption of SPDA along with an alternative mechanism TTC.
- 2003: Abdulkadiroğlu and Sönmez joined forces with Alvin Roth (Harvard) and his then student Parag Pathak to convince some major U.S. cities to adopt SPDA.
- 2003: New York City adopted SPDA for high school admissions.
- 2005: Boston adopted SPDA for K-12 admissions.
- 2007: SPDA adopted throughout England.
- 2007-present: Several other school districts adopted SPDA or TTC.

Subsequent Developments: 2012 Nobel Prize

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Press release

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The Prize in Economic Sciences 2012

The Royal Swedish Academy of Sciences has decided to award the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel for 2012 to

Alvin E. Roth
Harvard University, Cambridge, MA, USA, and
Harvard Business School, Boston, MA, USA

and

Lloyd S. Shapley
University of California, Los Angeles, CA, USA

"for the theory of stable allocations and the practice of market design".

The Role of Student Placement in 2012 Nobel Prize

- Student Placement along with kidney exchange research of Sönmez, Ünver and Roth played a key role in 2012 Economics Nobel.

"The work by Alvin Roth has enhanced our understanding of how markets work. Using empirical, experimental and theoretical methods, Roth and his coauthors, including A. Abdulkadiroğlu, P.A. Pathak, T. Sönmez and M.U. Ünver, have studied the institutions that improve market performance, thereby illuminating the need for stability and incentive compatibility. These contributions led directly to the successful redesign of a number of important real-world markets."

Nobel Prize Committee, October 15 2012

Conclusion

- Analysis of the Turkish student assignment mechanism by Balinski & Sönmez (1999) and U.S. school choice mechanisms by Abdulkadiroğlu & Sönmez (2003) initiated a literature on design of student assignment mechanisms.
- SPDA is extended by several authors to accommodate various considerations including reserves for specific groups of students (eg. minorities, valedictorians at schools, students who are willing to pay full tuition, etc.).

  SPDA continues to be well-behaved under these considerations.
- Several countries as well as school districts around the world adopted SPDA in the last decade due to its superior properties.
- Turkey can also benefit by adopting SPDA for University Admissions!