

# How Does Matching Theory Improve Our Lives?

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# Overview


- In 1962 David Gale and Lloyd Shapley published one of the most influential papers in game theory starting the literature in matching theory.
- Until early 2000s the main practical application of matching theory was entry level labor markets such as the U.S. hospital-intern market.
- This trend has recently changed as matching theory found new applications in (often large scale) resource allocation problems of social importance.
- Research on two of these new applications has especially been influential in practice:
  - School Choice
  - Kidney Exchange

# Kidney Exchange

- Shortly after the first few matching papers on kidney exchange are published in 2004-2005, several regions in the U.S. and a few countries launched centralized kidney exchange programs.
- In his June 2011 Congress testimony, Dr. Myron Gutmann, Assistant Director at NSF emphasized, research on kidney exchange has resulted in measurable gains for the U.S. taxpayer.
- Similarly, in a recent NSF - Science Nation story, Nancy Lutz, program director at NSF remarked:


“In addition, it’s especially rewarding to see such a clear and immediate benefit to the public. This research moved from abstract, academic theory to real world, direct impact very quickly.”

# Kidney Exchange



## Bringing People Into Focus

How Social, Behavioral and Economic Research Addresses National Challenges



National Science Foundation

## OPTIMIZING THE USE OF VITAL RESOURCES

Applying methods from the field of economics to topics ranging from public health to communication to education, NSF-supported research is helping answer questions about a wide range of important problems not commonly considered to be economic issues.



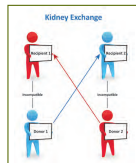
### Streamlining Kidney Transplant Exchanges

More than 65,000 kidney transplants are conducted each year. Finding suitable kidney donors for those in need of a transplant has long been a daunting challenge for both anxious recipients and the medical establishment.

To address the high demand for kidneys and the challenge of finding a donor, economists have developed algorithms to facilitate kidney matching for patients who have willing but biologically incompatible donors. Based on their knowledge in game theory and market dynamics, Alvin Roth of Harvard University, Tayfun Sonmez of Boston College and M. Utku Ünver of the University of Pittsburgh developed powerful match-making software that optimizes the process of identifying an appropriate live donor match with compatible blood types and antibodies.

This system creates kidney exchanges that match an incompatible donor-patient pair with a similarly incompatible pair so that each of the patients receives a kidney from a compatible donor. The medical programs that use this software have already saved many lives nationwide. The researchers are now investigating the increased efficiency between two-way and three-way matches, as well as more extended transplant chains.

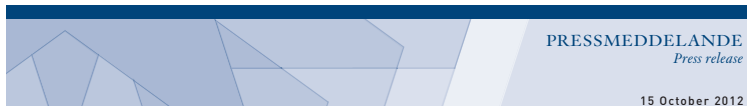
Alvin Roth was a co-recipient of the 2012 Nobel Prize in Economic Sciences for his research on the practical applications of matching theory.



# School Choice

- Shortly after the first matching paper on school choice is published in 2003, several school districts adopted mechanisms advocated in this paper. These school districts include:
  - New York City
  - Boston
  - Chicago
  - Denver
  - New Orleans
- Perhaps more strikingly, these mechanisms are adopted throughout England by all local authorities (more than 150 of them) by 2007.

# Ultimate Recognition: 2012 Nobel Prize



## The Prize in Economic Sciences 2012

The Royal Swedish Academy of Sciences has decided to award the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel for 2012 to

**Alvin E. Roth**

and

**Lloyd S. Shapley**

Harvard University, Cambridge, MA, USA, and  
Harvard Business School, Boston, MA, USA

University of California, Los Angeles, CA, USA

*"for the theory of stable allocations and the practice of market design".*



# What is Happening?

- The catalyst for the recent success of matching theory has been the strong theory tradition pioneered by Gale & Shapley (*AMM* 1962) coupled with careful modeling bringing the theory and the practice closer.
- Prior to 1990s, research on matching was mostly focused on two-sided matching markets.

The influential monograph of Roth & Sotomayor (1990) gives a very clear picture of the focus of matching literature in this era.

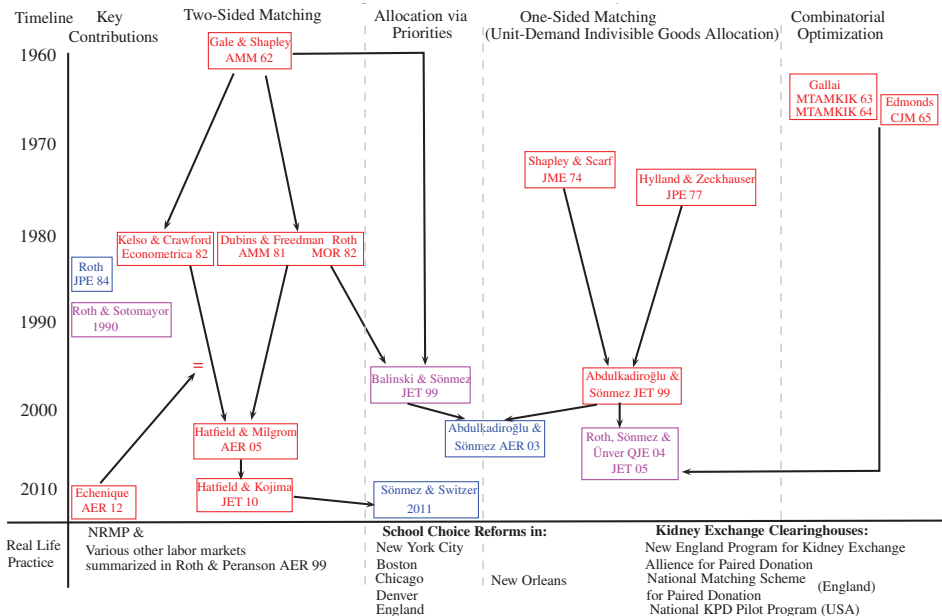
- While Shapley & Scarf (*JME* 1974) and Hylland & Zeckhauser (*JPE* 1977) introduced two early models of “one-sided matching,” these models received much less attention until the late 1990s.

# The Rise of One-Sided Matching

- This trend has changed considerably starting with late 1990s. Not only the focus on one-sided matching models increased, matching theorists developed new models at the interface of one-sided matching and two-sided matching.
- It is this recent trend which eventually lead to a number of new applications, including school choice and kidney exchange.
- The last few years have also seen significant advances in two-sided matching theory with the introduction of Hatfield & Milgrom (*AER* 2005) matching with contracts model.
- Having learned from past experience, market designers have immediately explored the potential links of this exciting model with one-sided matching.

This approach has already resulted in a brand new application of matching theory: Cadet-branch matching.





# Outline of Today's Lecture

- Thanks to Roth & Sotomayor (1990), the two-sided matching literature prior to 1990s is well-known.
- Today, I will focus on some of the major subsequent developments presented as simple case studies.
  - On-Campus Housing
  - Kidney Exchange
  - Assignment of Students to Schools

# On-Campus Housing

- A number of houses should be allocated to a group of agents with the following popular real-life mechanism **Random Serial Dictatorship**:
  1. Participants are ordered in a queue with an even lottery.
  2. They submit their strict preferences over houses.
  3. The first agent in queue is assigned his first choice; the second agent is assigned his first choice among remaining houses, and so on.
- One of the houses is occupied and its tenant is given two options:
  1. To keep his current house, or
  2. to give it up and enter the "lottery."
- Since there are no guarantees to get a better house, the existing tenant may choose the first option which in turn may result in loss of potential gains from trade.

## An Example

There are three agents  $i_1, i_2, i_3$  and three house  $h_1, h_2, h_3$ . Agent  $i_1$  is a current tenant and he occupies house  $h_1$ . Agents  $i_2, i_3$  are new applicants and house  $h_2, h_3$  are vacant houses.

- Utilities are given as follows:

	$h_1$	$h_2$	$h_3$
$i_1$	3	4	1
$i_2$	4	3	1
$i_3$	3	4	1

- Agent  $i_1$  has two options:
  - he can keep house  $h_1$  or
  - he can give it up and enter the lottery.
- His utility from keeping house  $h_1$  is 3.

## An Example

- The following table summarizes the possible outcomes, in case he enters the lottery:

ordering	$i_1$	$i_2$	$i_3$
$i_1 - i_2 - i_3$	$h_2$	$h_1$	$h_3$
$i_1 - i_3 - i_2$	$h_2$	$h_3$	$h_1$
$i_2 - i_1 - i_3$	$h_2$	$h_1$	$h_3$
$i_2 - i_3 - i_1$	$h_3$	$h_1$	$h_2$
$i_3 - i_1 - i_2$	$h_1$	$h_3$	$h_2$
$i_3 - i_2 - i_1$	$h_3$	$h_1$	$h_2$

- Expected utility from entering the lottery:

$$\frac{1}{6}u(h_1) + \frac{3}{6}u(h_2) + \frac{2}{6}u(h_3) = \frac{3}{6} + \frac{12}{6} + \frac{2}{6} = \frac{17}{6} < 3.$$

- Optimal strategy:** Keeping house  $h_1$ .

# An Example

When agent  $i_1$  keeps house  $h_1$ :

- Since both  $i_2, i_3$  prefer  $h_2$  to  $h_3$ , the eventual outcome is either

$$\begin{pmatrix} i_1 & i_2 & i_3 \\ h_1 & h_2 & h_3 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} i_1 & i_2 & i_3 \\ h_1 & h_3 & h_2 \end{pmatrix}$$

both with  $1/2$  probability.

- Inefficiency:** The first outcome is Pareto dominated by

$$\begin{pmatrix} i_1 & i_2 & i_3 \\ h_2 & h_1 & h_3 \end{pmatrix}$$

## Avoiding Inefficiency with One Existing Tenant

The cause for the inefficiency is the lack of the mechanism to guarantee the existing tenant a house that is at least as good as the one he already holds. One natural modification that will fix this “deficiency” is the following:

1. Order the agents with a lottery.
2. Assign the first agent his top choice, the second agent his top choice among the remaining houses, and so on, **until someone demands the house the existing tenant holds.**
3.
  - a. If the existing tenant is already assigned a house, then do not disturb the procedure.
  - b. If the existing tenant is not assigned a house, then modify the remainder of the ordering by inserting him to the top, and proceed with the procedure.

# A General Model: House Allocation with Existing Tenants

We can generalize this simple example with the following **House Allocation with Existing Tenants** model (Abdulkadiroğlu & Sönmez *JET* 1999):

- A set of houses should be allocated to a set of agents by a centralized clearing house.
- Some of the agents are existing tenants each of whom already occupies a house and the rest of the agents are newcomers.
- In addition to occupied houses, there are vacant houses.
- Existing tenants are not only entitled to keep their current houses but also apply for other houses.



## Popular Real-Life Mechanism: RSD with Squatting Rights

- Each existing tenant decides whether she will enter the housing lottery or keep her current house. Those who prefer keeping their houses are assigned their houses. All other houses become available for allocation.
- An ordering of agents in the lottery is randomly chosen from a given distribution of orderings. This distribution may be uniform or it may favor some groups.
- Once the agents are ordered, available houses are allocated using the induced **simple serial dictatorship**: The first agent receives her top choice, the next agent receives her top choice among the remaining houses and so on so forth.

**Major deficiency:** Neither **individually rational** nor **Pareto efficient**.

## Solution: You Request My House - I Get Your Turn (YRMH-IGYT) Mechanism

1. For any given ordering, assign the first agent his top choice, the second agent his top choice among the remaining houses, and so on, until someone demands the house of an existing tenant.
2. If at that point the existing tenant whose house is demanded is already assigned a house, then do not disturb the procedure.  
Otherwise modify the remainder of the ordering by inserting him to the top and proceed with the procedure.
3. Similarly, insert any existing tenant who is not already served at the top of the line once his house is demanded.
4. If at any point a **cycle** forms, it is formed by exclusively existing tenants and each of them demands the house of the tenant next in the cycle.

In such cases remove all agents in the cycle by assigning them the houses they demand and proceed with the procedure.

# Efficiency, Individual Rationality, and Strategy-Proofness

**Theorem:** For any ordering, the YRMH-IGYT mechanism is:

1. individually rational,
2. Pareto efficient, and
3. strategy-proof.

# Example: Mechanics of the YRMH-IGYT Mechanism

Existing Tenants:  $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9$

Occupied Houses:  $h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9$

Newcomers:  $a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}$

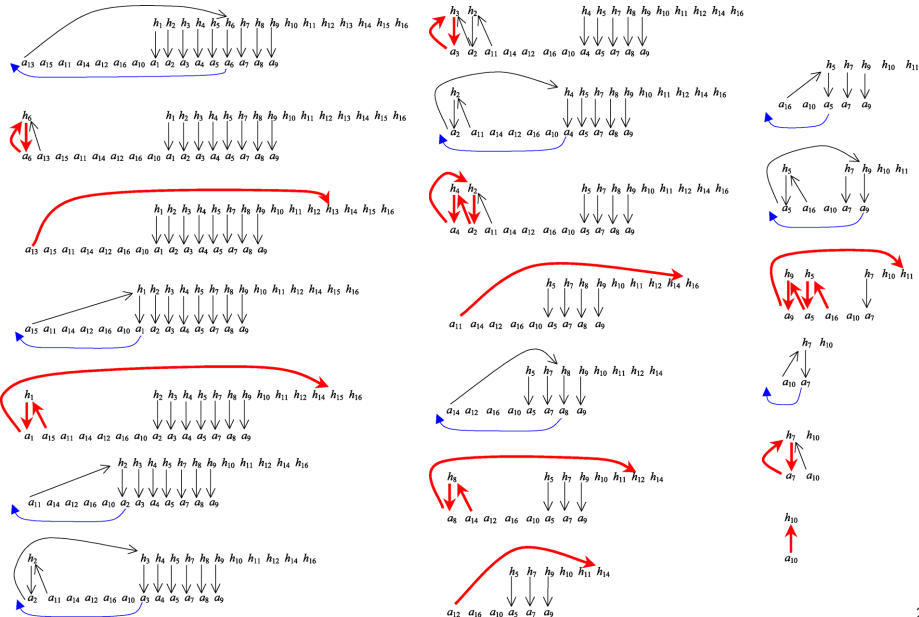
Vacant Houses:  $h_{10}, h_{11}, h_{12}, h_{13}, h_{14}, h_{15}, h_{16}$

Preferences:

$A_E$									$A_N$						
$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	$a_6$	$a_7$	$a_8$	$a_9$	$a_{10}$	$a_{11}$	$a_{12}$	$a_{13}$	$a_{14}$	$a_{15}$	$a_{16}$
$h_{15}$	$h_3$	$h_1$	$h_2$	$h_9$	$h_6$	$h_6$	$h_6$	$h_{11}$	$h_7$	$h_2$	$h_4$	$h_6$	$h_8$	$h_1$	$h_5$
$\vdots$	$h_4$	$h_3$	$\vdots$	$\vdots$	$\vdots$	$h_7$	$h_{12}$	$\vdots$	$h_3$	$h_4$	$h_{14}$	$h_{13}$	$\vdots$	$\vdots$	$\vdots$
	$\vdots$	$\vdots$				$\vdots$	$\vdots$		$h_{12}$	$h_{16}$	$\vdots$	$\vdots$			
									$h_{10}$	$\vdots$					
									$\vdots$						

Lottery Order:  $a_{13} a_{15} a_{11} a_{14} a_{12} a_{16} a_{10} a_1 a_2 a_3 a_4 a_5 a_6 a_7 a_8 a_9$

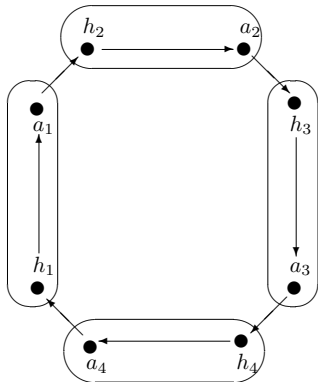
# Red Arrows: Finalized Assignments



## Cycles and Chains

Observe that YRMH-IGYT mechanism allocates houses via two different ways:

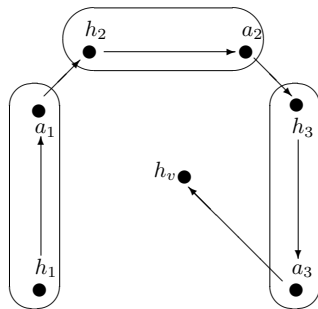
1. **Cycles:** The first type of transaction is exclusively between existing tenants and it is simply a **house swap!**



# Cycles and Chains

2. **Chains:** Under the second type of transaction,

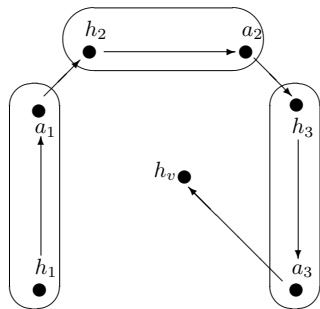
- the agent at the top of the chain receives an available house (i.e. either a vacant house or an occupied house whose occupant is already served),
- the next agent receives the house of the preceding agent in the chain, and so on...



# Cycles and Chains

2. **Chains:** Under the second type of transaction,

- the agent at the top of the chain receives an available house (i.e. either a vacant house or an occupied house whose occupant is already served),
- the next agent receives the house of the preceding agent in the chain, and so on...



*Who would have thought, these cycles and chains would save hundreds of lives every year, shortly after YRMH-IGYT mechanism was introduced!*



# Reinterpretation of the Model

Suppose

- **existing tenants** are reinterpreted as **kidney patients** with incompatible donors,
- **occupied houses** are reinterpreted as **incompatible donors** of kidney patients,
- **newcomers** are reinterpreted as **kidney patients** with no donors, and
- **vacant houses** are reinterpreted as either **good-samaritan donors** who donate their kidneys to unspecified strangers, or as **deceased-donor kidneys** “borrowed” from the deceased-donor queue.

# Evolution of Cycles & Chains to Kidney Cycles & Chains

- Optimizing the potential of **kidney exchanges** by organizing cycles and chains observed in YRMH-IGYT mechanism provided the backbone of Roth, Sönmez & Ünver (*QJE* 2004).
- Today we already see the practical impact of this research program in several countries!



# Kidney Transplants

- There are close to 96,000 patients on the waiting list for cadaver kidneys in the U.S. as of April 2013.
- In 2012:
  - 34,840 patients were added to waiting list while 28,437 patients were removed;
  - 10,868 transplants of deceased donor kidneys performed; and
  - 4,185 patients died while on the waiting list and 2,667 were removed from the list due to being too sick to receive a transplant.
  - There were also 5,619 transplants of kidneys from **living** donors.
- Often living donors are **incompatible** with their intended patient.

## Institutional Constraint: No Money

- The shortage of kidney increases by about 3,500 kidneys each year in the U.S.
- The 1984 National Organ Transplant Act (and in many states the Uniform Anatomical Gift Act) makes paying for an organ for transplantation a felony.

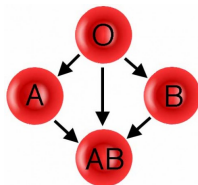
Section 301, National Organ Transplant Act (NOTA), 42 U.S.C. 274e 1984:

“it shall be unlawful for any person to knowingly acquire, receive or otherwise transfer any human organ for valuable consideration for use in human transplantation.”

- There is a rich literature on whether the ban on buying and selling of kidneys be repealed (ex: Becker & Elias *JEP* 2007).

## Medical Constraint: ABO Blood Type Compatibility

- There are four blood types: A, B, AB and O.

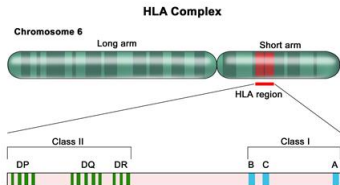


- In the absence of other complications:
  - Type O kidneys can be transplanted into any patient;
  - type A kidneys can be transplanted into type A or type AB patients;
  - type B kidneys can be transplanted into type B or type AB patients;
  - type AB kidneys can only be transplanted into type AB patients.
- Type O patients are disadvantaged because of this “natural injustice.”

## Medical Constraint: Tissue Type Compatibility

- Tissue type or **Human Leukocyte Antigen (HLA)** type: Combination of several pairs of antigens on Chromosome 6.

HLA proteins A, B, and DR are especially important.

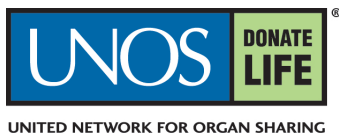


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- Prior to transplantation, the potential recipient is tested for the presence of preformed antibodies against donor HLA.  
If there is a **positive crossmatch**, the transplantation cannot be carried out.

## Allocation of Deceased Donor Kidneys in the U.S.

- U.S. Congress views deceased donor kidneys offered for transplantation as a national resource, and the 1984 NOTA established the [Organ Procurement and Transplantation Network \(OPTN\)](#).



- [United Network for Organ Sharing \(UNOS\)](#), as the OPTN contractor, overseas the allocation of deceased donor kidneys.

# Live Donor Transplants: Much Less Organized Until 2004

- A patient identifies a willing donor and, if the transplant is feasible, it is carried out.
- Otherwise, the patient remains on the queue for a cadaver kidney, while the donor returns home.
- Prior to emergence of organized kidney exchange programs in 2004, additional possibilities have been utilized in a small number of cases:
  - **Paired exchanges:** Exchanges between two incompatible pairs.
  - **Indirect exchanges:** An exchange between an incompatible pair and the deceased-donor queue.



# Paired Kidney Exchange

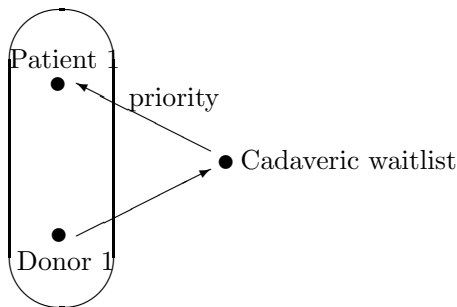


- First proposed by Rapaport (Transplantation Proceedings 1986).
- The first kidney exchanges were carried out in South Korea in early 1990s.
- Renewed interest in the U.S. with Ross et al. (NEJM 1997) on "Ethics of Kidney Exchange."

# Paired Kidney Exchange

- In 2000 the transplantation community issued a **consensus statement** declaring it as “ethically acceptable.”
- The consensus statement also specified the following **Incentives**  
**Constraint:** All four operations shall be carried out simultaneously!
- The first kidney exchange in the U.S. was carried out in Rhode Island in 2000.
- Prior to formal organized kidney exchange clearinghouses, very rare: 5 paired exchanges in New England between 2000-2004.

# Indirect Kidney Exchange



- Widespread concern in transplantation community: Indirect exchanges can harm type O patients with no living donors.
- Nevertheless, many transplant centers have started pilot indirect exchange programs since 2000 (ex: Johns Hopkins Comprehensive Transplant Center, New England Medical Center.)

# Kidney Exchange as a Market Design Problem

- In the early 2000s, we observed that the two main types of kidney exchanges conducted in the U.S. correspond to the most basic forms of transactions in [house allocation with existing tenants](#) model of Abdulkadiroğlu & Sönmez (*JET* 1999).
- Inspired by this observation and building on the existing practices in kidney transplantation, we analyzed in Roth, Sönmez, & Ünver (*QJE* 2004) how an efficient and incentive-compatible system of exchanges might be organized, and what its welfare implications might be.

- Prior to our interaction with the transplantation community, three assumptions shaped our initial modeling of kidney exchange:
  1. Patient preferences over compatible kidneys.
    - a. The “European” view: The graft survival rate increases as the tissue type mismatch decreases (Opelz *Transplantation* 1997).
    - b. The “American” view: The graft survival rate is the same for all compatible kidneys (Gjertson & Cecka *Kidney International* 2000, Delmonico *NEJM* 2004).
  2. The number of simultaneous transplants.
  3. Feasibility of indirect exchanges.
- In subsequent analysis, a few other factors also proved to be important:
  4. Integration of good-samaritan donors (a.k.a. altruistic donors).  
Sequential implementation of good-samaritan chains.
  5. Participation by compatible pairs.
  6. Center Incentives.
  7. Dynamic aspects.

# First Pass: RSÜ (QJE 2004), “Kidney Exchange”

- **Assumption 1:** The graft survival rate increases as the tissue type mismatch decreases (i.e. the European view).
- **Assumption 2:** There is no constraint on the number of transplants that can be simultaneously carried out.
- **Assumption 3:** Indirect exchanges are feasible.
- This first kidney exchange model builds on **house allocation with existing tenants** model of Abdulkadiroğlu & Sönmez (*JET* 1999) and advocates the **top trading cycles and chains (TTCC)** mechanism, a generalization of the YRMH-IGYT mechanism.
- **Other Related Literature:**
  - Shapley & Scarf (*JME* 1974)
  - Roth & Postlewaite (*JME* 1977)
  - Roth (*Economics Letters* 1982)

# Simulations on Welfare Gains

NUMBER OF TRANSPLANTS AND QUALITY OF MATCH FOR N = 30, N = 100, AND N = 300

Pop. size	Pref.	Exchange regime	Total trans. %	Own donor trans. %	Trade %	Wait-list upgrade %	HLA mis.
n = 30	Wait-list 0%						
	All	None	54.83 (8.96)	54.83 (8.96)	0 (0)	0 (0)	4.79 (0.25)
	All	Paired	68.50 (9.90)	54.83 (8.96)	13.67 (9.40)	0 (0)	4.78 (0.24)
	Rational	TTC	82.47 (10.14)	23.03 (9.44)	59.43 (13.57)	0 (0)	4.16 (0.22)
	Cautious	TTC	81.07 (10.02)	34.17 (11.27)	46.90 (13.96)	0 (0)	4.29 (0.23)
	Wait-list 40%						
	All	Paired/ind.	68.50 (9.90)	54.83 (8.96)	13.67 (9.40)	13.20 (6.73)	4.78 (0.24)
	Rational	TTCC e	84.70 (8.49)	21.23 (9.60)	63.47 (12.39)	6.37 (4.88)	4.17 (0.22)
	Cautious	TTCC e	83.57 (8.53)	32.93 (10.98)	50.63 (12.54)	6.13 (4.39)	4.29 (0.22)
n = 100	Wait-list 0%						
	All	None	54.79 (4.48)	54.79 (4.48)	0 (0)	0 (0)	4.83 (0.14)
	All	Paired	73.59 (4.97)	54.79 (4.48)	18.80 (3.81)	0 (0)	4.82 (0.11)
	Rational	TTC	87.85 (4.54)	11.51 (3.44)	76.34 (5.45)	0 (0)	3.72 (0.10)
	Cautious	TTC	87.23 (4.73)	24.01 (4.48)	63.22 (5.46)	0 (0)	3.86 (0.11)
	Wait-list 40%						
	All	Paired/ind.	73.59 (4.97)	54.79 (4.48)	18.80 (3.81)	10.24 (3.07)	4.82 (0.11)
	Rational	TTCC e	89.44 (3.85)	10.29 (3.26)	79.15 (4.40)	3.96 (1.97)	3.71 (0.10)
	Cautious	TTCC e	88.97 (4.17)	22.81 (4.83)	66.16 (4.79)	4.72 (2.60)	3.85 (0.11)
n = 300	Wait-list 0%						
	All	None	53.92 (2.82)	53.92 (2.82)	0 (0)	0 (0)	4.81 (0.08)
	All	Paired	75.03 (2.72)	53.92 (2.82)	21.11 (2.51)	0 (0)	4.81 (0.07)
	Rational	TTC	91.05 (3.35)	5.72 (1.28)	85.32 (3.61)	0 (0)	3.29 (0.06)
	Cautious	TTC	90.86 (3.31)	15.36 (2.20)	75.51 (4.07)	0 (0)	3.40 (0.06)
	Wait-list 40%						
	All	Paired/ind.	75.03 (2.72)	53.92 (2.82)	21.11 (2.51)	9.77 (1.73)	4.81 (0.07)
	Rational	TTCC e	92.29 (2.98)	5.00 (1.29)	87.29 (3.05)	3.02 (1.36)	3.29 (0.06)
	Cautious	TTCC e	92.17 (2.93)	14.42 (2.10)	77.75 (3.26)	3.19 (1.40)	3.39 (0.06)

# Initial Reactions of the Transplantation Community

- Following RSÜ (*QJE* 2004), we entered into discussions with New England transplant surgeons and their colleagues in the transplant community.
- In the course of those discussions it became clear that a likely first step will be to implement logistically simpler **pairwise** exchanges.
- Furthermore, doctors indicated that they would be more comfortable with a model where patient preferences are assumed to be indifferent among all compatible kidneys.
- Finally doctors showed less interest in indirect exchanges due to concerns over blood-type O patients w/o living donors.
- This motivated RSÜ (*JET* 2005), “Pairwise Kidney Exchange.”



## Model 2: RSÜ (*JET* 2005), “Pairwise Kidney Exchange”

- **Assumption 1:** The graft survival rate is the same for all compatible kidneys (i.e. the American view).
- **Assumption 2:** No more than two transplants can be carried out simultaneously.
- **Assumption 3:** Indirect exchanges are not allowed.
- **Related Literature in Operations Research and Economics:**
  - Gallai (MTAMKIK 1963, 1964)
  - Edmonds (Can. J. of Math. 1965)
  - Bogomolnaia & Moulin (Econometrica 2004)

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  - Gallai (MTAMKIK 1963, 1964)
  - Edmonds (Can. J. of Math. 1965)
  - Bogomolnaia & Moulin (Econometrica 2004)
- The methodology, mechanisms, and techniques advocated in this paper provided the backbone of optimization-based kidney exchange programs in the U.S. and elsewhere.

## Subsequent Research on Kidney Exchange

- Despite the elegance of the underlying math and the presence of well-behaved mechanisms for pairwise kidney exchange, there is significant welfare gap between TTCC and efficient pairwise kidney exchange mechanisms.
- Two important factors in this welfare difference are:
  1. the loss of compatible pairs under pairwise exchange with dichotomous preferences; and
  2. the two-way exchange constraint.

## Subsequent Research on Kidney Exchange

- Hence we focused on increasing welfare in subsequent research:
  - RSÜ (*AER* 2007): Welfare gains from 3-way exchange is especially important.
  - Roth et. al (*AJT* 2006): “Simultaneous transplant” constraint can be relaxed for **good-samaritan donor chains** (a.k.a. **nondirected-donor chains**), and thus substantially larger exchanges can be conducted.
  - Sönmez & Ünver (2011): The impact of inclusion of **compatible pairs** in kidney exchange pool.
- While the transplantation community was initially hesitant about each of these design proposals, the first two became widespread by now.
- As for the third, there are promising developments: Columbia University (NYC, New York) and Methodist Specialty and Transplant Hospital (San Antonio, Texas) have adopted programs with compatible pairs.

## Collaboration with Transplantation Community

- **New England Program for Kidney Exchange (NEPKE):** Together with members of New England transplantation community, we have launched centralized kidney exchange in New England in 2004.

NEPKE became the first kidney exchange program to utilize optimization techniques.

- **Alliance for Paired Donation (APD):** We have also provided software support for the Alliance for Paired Donation.

APD currently has more than 80 transplant centers and it became the first program to heavily utilize non-simultaneous nondirected-donor chains we proposed in Roth et. al (*AJT* 2006).

# New England Program For Kidney Exchange

A transplant option for patients with an incompatible living donor

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## Welcome

### A Life-Saving Option

The New England Program for Kidney Exchange offers new life-saving options to those seeking a kidney transplant, but whose potential living donor is not a good biological "match" due to either blood type incompatibility or cross-match incompatibility. This option is known as kidney exchange, kidney paired donation, or kidney swap.

NEPKE uses a computer program to find cases where the donor in an incompatible pair can be matched to a recipient in another pair. By exchanging donors, a compatible match for both recipients may be found. You can learn more about the program [HERE](#) and read our [newsletter here](#).

NEPKE can also find potential kidney recipients for those generous people who seek to become non-directed living donors (otherwise known as Good Samaritan Donors or Altruistic Donors). Information about that process is available [HERE](#).

**NEWS:** Transplant centers are being provided with brochures to provide information about this program to their kidney patients.

[More News](#)

**NEPKE**  
Transplants  
to Date  
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**Notes:** There are many good websites on the Internet that help kidney patients learn more about transplant options.

[Links](#)

# Alliance for Paired Donation



Serving Kidney Patients through Technology, Education, and Generosity.

- ▶ Home
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- ▶ Patient Information
- ▶ Kidney Donor Info
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We have alliance partners in many locations  
Click map to view

Also visit  
[www.thenead.com](http://www.thenead.com)



Click here to view the People Magazine story on NEAD chains.

**I NEED A KIDNEY TRANSPLANT**

**I WOULD LIKE TO DONATE A KIDNEY**

**I WANT TO DONATE FINANCIALLY**

## Alliance for Paired Donation – Saving Lives through Kidney Paired Donation

More than 84,000 people in America are waiting for a kidney transplant; sadly, about 12 of these patients die every day because there aren't enough donors. Many kidney patients have someone who is willing to donate, but because of immune system or blood type incompatibilities, they are not able to give a kidney to their loved one.

The Alliance for Paired Donation can help. Kidney paired donation matches one incompatible donor/recipient pair to another pair in the same situation, so that the donor of the first pair gives to the recipient of the second, and vice versa. In other words, the two pairs swap kidneys. APD has also pioneered a new way of using altruistic, or good Samaritan, donors, so that the transplants no longer have to be performed simultaneously. Non-simultaneous Extended Altruistic Donor Chains (NEAD Chains) allow donors to "pay it forward" after their loved one receives a transplant.



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Paying it Forward:  
Saving Lives Through Paired  
Kidney Exchange  
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2009 Annual Report

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# Amendment of National Organ Transplant Act (NOTA)

- When we initially helped found NEPKE, it was unclear whether kidney exchange is in violation of NOTA.
- In particular, it was unclear whether kidney exchange was considered to involve transfer of a human organ for **valuable consideration**.
- In Dec 2007, an amendment of NOTA has passed in the U.S. Senate, clarifying that kidney exchange is legal.
- **Charlie W. Norwood Living Organ Donation Act**, opened the doorway for national kidney exchange in the U.S.

## One Hundred Tenth Congress of the United States of America

AT THE FIRST SESSION

*Began and held at the City of Washington on Thursday,  
the fourth day of January, two thousand and seven*

### An Act

To amend the National Organ Transplant Act to provide that criminal penalties do not apply to human organ paired donation, and for other purposes.

*Be it enacted by the Senate and House of Representatives of  
the United States of America in Congress assembled,*

#### SECTION 1. SHORT TITLE.

This Act may be cited as the "Charlie W. Norwood Living Organ Donation Act".

#### SEC. 2. AMENDMENTS TO THE NATIONAL ORGAN TRANSPLANT ACT.

Section 301 of the National Organ Transplant Act (42 U.S.C. 274e) is amended—

(1) in subsection (a), by adding at the end the following:

"The preceding sentence does not apply with respect to human organ paired donation."; and

(2) in subsection (c), by adding at the end the following:

"(4) The term 'human organ paired donation' means the donation and receipt of human organs under the following circumstances:

"(A) An individual (referred to in this paragraph as the 'first donor') desires to make a living donation of a human organ specifically to a particular patient (referred to in this paragraph as the 'first patient'), but such donor is biologically incompatible as a donor for such patient.

"(B) A second individual (referred to in this paragraph as the 'second donor') desires to make a living donation of a human organ specifically to a second particular patient (referred to in this paragraph as the 'second patient'), but such donor is biologically incompatible as a donor for such patient.

"(C) Subject to subparagraph (D), the first donor is biologically compatible as a donor of a human organ for the second patient, and the second donor is biologically compatible as a donor of a human organ for the first patient.


"(D) If there is any additional donor-patient pair as described in subparagraph (A) or (B), each donor in the group of donor-patient pairs is biologically compatible as a donor of a human organ for a patient in such group.

"(E) All donors and patients in the group of donor-patient pairs (whether 2 pairs or more than 2 pairs) enter into a single agreement to donate and receive such human organs, respectively, according to such biological compatibility in the group.



# National Kidney Exchange in the U.K.

2009: RSÜ (2005, 2007) provided the basis for national kidney exchange in UK where a group of computer scientists at U. of Glasgow helped design the **National Matching Scheme for Paired Donation**. Their algorithm finds an optimal matching under 2-way + 3-way exchanges.


**organ donation**

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## Paired donation matching scheme

### The matching process


All incompatible couples wishing to pursue the option of paired donation must be registered with NHSBT (ODT) for the national matching scheme. 'Matching runs' will be carried out at intervals determined by the rate at which incompatible couples join the paired donation list. There are three key stages in the matching process to identify suitable pairs for transplant.

Firstly, all matching pairs must be identified. Secondly, all possible combinations of these pairs must be determined. The final stage is then to identify the optimal combination of matching pairs from all of those possible. It is important to consider not only which couples are involved but also the nature of the exchanges involved (ie who donates to whom). This part of the process is carried out using optimal matching algorithms, in collaboration with a team of researchers from the University of Glasgow (for further details see <http://www.optimalmatching.com>)

A number of criteria have been agreed to identify the most appropriate combination of matching pairs. These are local exchange, sensitisation, HLA match, age difference between two donors in a matching pair (donor-donor age difference) and blood group match.

### The National Matching Scheme

A scoring system has been agreed to identify the most suitable combination of matching pairs using the five criteria above. It provides a compromise between the competing objectives of the matching scheme. A score is calculated for each of the two potential transplants in a matching pair. For each possible combination of matching pairs a total score is then calculated and the highest scoring combination identifies the set of proposed transplants.



**Join the Organ Donor Register**  
**0300 123 23 23**  
[Amend your details](#)  
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**In this section**

- [Arrangements for altruistic non-directed living kidney donation](#)
- [Altruistic non-directed living kidney donation - summary](#)
- [Arrangements for paired/pooled living kidney donation](#)
- [Paired/pooled living kidney donation - summary](#)
- [Paired/pooled donation matching scheme](#)

**2010:** A pilot national kidney exchange program in U.S. is launched, also adopting an optimal mechanism under 2-way + 3-way exchanges.

As of December 2011, NEPKE is part of the national kidney exchange pilot program.

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# Property Rights

- The structure of **property rights** for the goods to be allocated plays a central role in determining which mechanisms can be potentially used.
- So far, both for house allocation and kidney exchange, we have seen two types of property rights:
  - **Private Ownership:**
    - Existing tenants have “full control” of the houses they occupy in the sense that they could keep them or trade them.
    - Kidney patients with willing donors cannot be asked to part from their donors unless they wish to do so.
  - **Collective Ownership:**
    - All agents, in principle, have the same claims over vacant houses.
    - All patients, in principle, have the same claims over kidneys donated by non-directed donors.

# Priority Lists

- Sharing goods when there is collective ownership is a challenging task.
- One popular method, especially when goods are indivisible is, based on determining property rights via **priority lists**.
- Under this approach, for a given object  $s$  agents are priority ordered in a list  $\pi_s$ , so that, an agent  $i$  who appears earlier in the list than another agent  $j$  has higher claims on object  $s$  than agent  $j$ .

It is convenient to interpret the priority list  $\pi_s$  as a “queue” for  $s$ .

- Depending on the application, the priority list might be the same for some or all of the objects.
- In some applications there might be a natural hierarchy which determines the priority lists; in others priority lists may be random.

# Allocation with a Uniform Priority List

- For many applications, there is a natural priority list that is uniform across all goods.
  - Allocation of public school seats via a uniform standardized test.
  - Branch allocation in the Army via Order of Merit List.
- **Simple serial dictatorship (SSD)** is a natural mechanism to use in such applications: The first agent is allocated his top choice, the next allocation is allocated his top choice among remaining choices, etc.
- A mechanism **eliminates justified envy** if it never assigns a good to an agent at the expense of a higher priority agent.
- **Theorem** (Balinski & Sönmez *JET* 1999): The SSD induced by the uniform priority list is the only Pareto efficient mechanism which eliminates justified envy.
- As importantly, this intuitive mechanism is strategy-proof as well.

## Allocation via Multiple Priority Lists

- There are several applications where the priority list might differ between objects. This is a more challenging problem.
- **Key Observation:** These one-sided matching problems are closely related to two-sided matching problems with the following twist. Interpret each object as an agent whose preferences are determined by the related priority list!
- This observation by Balinski & Sönmez (*JET* 1999) has been instrumental in several recent school choice mechanism reforms starting with the reforms in NYC (2003) and in Boston (2005).
- Understanding failures of some of the major competing real-life mechanisms will help us to put these reforms into context.

## College Admissions in Turkey

- Admissions to college is centralized and via a nationwide exam in Turkey. Upon taking this exam, students are ranked in several priority lists where different weights on topics are used to construct each priority list.
- Each department of a university is associated with only one of these priority lists. The mapping of departments to priority lists is exogenously determined by the central planner.  
**Ex:** Engineering schools use the list with higher weight for math, medical schools use the list with higher weight for science, etc.
- Students submit their preferences over departments to central planner after learning their places in each of the priority lists.

## Turkish College Admissions Mechanism

Since there is more than one priority list, SSD cannot be directly used to allocate college seats in Turkey. To handle the complication, Turkish officials came up with the following iterative application of multiple SSDs:

- 1a. Partition departments based on their associated priority list.

Tentatively allocate seats in each “category” with the resulting SSD. Observe that students might get multiple tentative assignments at this point.

- 1b. **Truncate** preferences of each student right after their highest ranked tentative assignment, provided that they have at least one.

Step 1b assures that no student gets multiple slots.

- Repeat Steps 1a,b with truncated preferences until no student receives multiple tentative assignments.

Terminate the procedure and finalize the assignments at this point.



## Example:

Students	=	{Alp, Banu, Can, Derin, Elif}
Colleges	=	{ $c_1$ , $c_2$ , $c_3$ }
College capacities	=	(2, 1, 1)
Skill Categories	=	{MF, TM}
$t(c_1)$	=	MF
$t(c_2) = t(c_3)$	=	TM

Student preferences and exam scores are as follows:

$R_A :$	$c_2 - c_1 - \emptyset$	$f^A =$	(450, 450)
$R_B :$	$c_1 - c_2 - c_3 - \emptyset$	$f^B =$	(400, 300)
$R_C :$	$c_1 - c_3 - c_2 - \emptyset$	$f^C =$	(350, 350)
$R_D :$	$c_1 - c_2 - \emptyset$	$f^D =$	(300, 400)
$R_E :$	$c_2 - c_3 - c_1 - \emptyset$	$f^E =$	(250, 250)

Note that these scores induce the following rankings in each category:

$$\pi_{\text{MF}} : A \ B \ C \ D \ E \qquad \pi_{\text{TM}} : A \ D \ C \ B \ E$$

## Step 1:

$$\pi_{MF} : \begin{array}{ccccc} A & B & C & D & E \\ c_1 & c_1 & & & \end{array}$$

$$\pi_{TM} : \begin{array}{ccccc} A & D & C & B & E \\ c_2 & - & c_3 & & \end{array}$$

Step 1 yields the following tentative student placement:

$$\nu^1 = \left( \begin{array}{ccccc} \text{Alp} & \text{Banu} & \text{Can} & \text{Derin} & \text{Elif} \\ c_1, c_2 & c_1 & c_3 & \emptyset & \emptyset \end{array} \right)$$

Having assigned at least one slot, preferences of students Alp, Banu, Can are truncated:

$$R_A^1 : c_2 - \emptyset$$

$$R_B^1 : c_1 - \emptyset$$

$$R_C^1 : c_1 - c_3 - \emptyset$$

For other students:  $R_D^1 = R_D$ , and  $R_E^1 = R_E$ .

**Step 2:** In Step 2 we first find the serial dictatorship outcomes for  $R^1$ .

$$\pi_{MF} : \begin{array}{ccccc} A & B & C & D & E \\ - & c_1 & c_1 & & \end{array} \quad \pi_{TM} : \begin{array}{ccccc} A & D & C & B & E \\ c_2 & - & c_3 & & \end{array}$$

Step 2 yields the following tentative student placement:

$$\nu^2 = \left( \begin{array}{ccccc} \text{Alp} & \text{Banu} & \text{Can} & \text{Derin} & \text{Elif} \\ c_2 & c_1 & c_1, c_3 & \emptyset & \emptyset \end{array} \right)$$

Having assigned two slots, preferences of student Can is truncated:

$$R_C^2 : c_1 - \emptyset$$

For other students:  $R_A^2 = R_A^1$ ,  $R_B^2 = R_B^1$ ,  $R_D^2 = R_D^1$ , and  $R_E^2 = R_E^1$ .

**Step 3:** In Step 3 we first find the serial dictatorship outcomes for  $R^2$ .

$$\pi_{\text{MF}} : \begin{array}{ccccc} \text{A} & \text{B} & \text{C} & \text{D} & \text{E} \\ - & c_1 & c_1 & & \end{array} \quad \pi_{\text{TM}} : \begin{array}{ccccc} \text{A} & \text{D} & \text{C} & \text{B} & \text{E} \\ c_2 & - & - & - & c_3 \end{array}$$

Step 3 yields the following tentative student placement (which is also a matching):

$$\nu^3 = \left( \begin{array}{ccccc} \text{Alp} & \text{Banu} & \text{Can} & \text{Derin} & \text{Elif} \\ c_2 & c_1 & c_1 & \emptyset & c_3 \end{array} \right)$$

Since no student is assigned more than one slot in  $\nu^3$ , the algorithm terminates resulting in:

$$\varphi^{\text{Turkish}}(R, f, q) = \nu^3$$

# Mechanisms via “Related” Two-Sided Matching Problem

- Recall that we can induce a two-sided matching problem for each priority-based allocation problem by pretending as if each object (school here) is an agent with preferences determined by the attached priority list.

This observation helps us to formulate the following alternative mechanisms for Turkish college admissions:

- **College-optimal stable mechanism (COSM):** The mechanism which selects the college-optimal stable matching of the associated two-sided matching problem for each problem.
- **Student-optimal stable mechanism (SOSM):** The mechanism which selects the student-optimal stable matching of the associated two-sided matching problem for each problem.

## Student-Proposing Deferred Acceptance Algorithm (Gale & Shapley *AMM* 1962)

The outcome of the SOSM can be obtained with the following algorithm.

**Step 1:** Each student proposes to her first choice. Each school tentatively assigns its seats to its proposers one at a time following their priority order. Any remaining proposers are rejected.

In general, at

**Step  $k$ :** Each student who was rejected in the previous step proposes to her next choice. Each school considers the students it has been holding together with its new proposers and tentatively assigns its seats to these students one at a time following their priority order. Any remaining proposers are rejected.

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The outcome of COSM can be obtained with an analogous algorithm where the roles of students and schools are reversed.

# An Equivalence

- **Theorem** (Balinski & Sönmez *JET* 1999):  
Turkish College Admissions Mechanism = COSM
- **Remark 1:** This result is reminiscent of the classical Roth (*JPE* 1982) result that shows the equivalence of the NRMP mechanism and the COSM.
- **Remark 2:** This equivalence is alarming given the following classical result from two-sided matching.  
**Theorem** (Gale & Shapley *AMM* 1962): Every student weakly prefers the outcome of SOSM to any stable matching including the outcome of COSM.
- **Bad Idea:** Using COSM in an environment where schools are objects!



# Pareto Efficiency

## Example:

**Students:**  $I = \{i_1, i_2\}$       **Colleges:**  $C = \{c_1, c_2\}$

**Capacities:** Both schools have 1 slot each

**Priority Lists:**  $(\pi_1, \pi_2)$  with  $c_1$  attached to list  $\pi_1$  and  $c_2$  to list  $\pi_2$

**Student preferences & exam scores:**

$$R_{i_1} : c_1 - c_2 \qquad f^{i_1} = (6, 8)$$

$$R_{i_2} : c_2 - c_1 \qquad f^{i_2} = (8, 6)$$

The algorithm terminates in one step resulting in the following Pareto inefficient matching:

$$\begin{pmatrix} i_1 & i_2 \\ c_2 & c_1 \end{pmatrix}$$

**Theorem (Gale & Shapley AMM 1962):** SOSM Pareto dominates any other mechanism that eliminates justified envy.

# Strategy-Proofness

**Example continued:** Recall that the outcome of the Turkish mechanism in the previous example was:

$$\begin{pmatrix} i_1 & i_2 \\ c_2 & c_1 \end{pmatrix}$$

Now suppose  $i_1$  announces a fake preference relation  $\tilde{R}_{i_1}$  where only  $c_1$  is acceptable. In this case the outcome changes as

$$\begin{pmatrix} i_1 & i_2 \\ c_1 & c_2 \end{pmatrix}$$

and hence student  $i_1$  successfully manipulates the Turkish mechanism.

- **Theorem** (Dubins & Freedman *AMM* 1981, Roth *MOR* 1982): SOSM is *strategy-proof*.
- A mechanism is **non-wasteful** if it never leaves an unmatched slot when there are students who would rather have it.
- **Theorem** (Alcalde & Barberà *ET* 1994): SOSM is the only mechanism that *eliminates justified envy*, and is *individually rational*, *non-wasteful*, and *strategy-proof*.

## Respecting Improvements

**Example further continued:** Recall that the outcome of the Turkish mechanism in the previous example was:

$$\begin{pmatrix} i_1 & i_2 \\ c_2 & c_1 \end{pmatrix}$$

Now suppose student  $i_1$  performs worse in both tests reflected by the following lower scores:  $\tilde{f}^{i_1} = (5, 5)$ . The outcome of the Turkish mechanism changes as follows under this alternative scenario:

$$\begin{pmatrix} i_1 & i_2 \\ c_1 & c_2 \end{pmatrix}$$

Ironically, student  $i_1$  is rewarded by getting his top choice as a result of a worse performance!

- A mechanism **respects improvements** if a student never receives a worse assignment as a result of an increase in one or more of his test scores.
- **Theorem** (Balinski & Sönmez *JET* 1999): SOSM respects improvements.
- **Theorem** (Balinski & Sönmez *JET* 1999): SOSM is the only mechanism that is *individually rational*, *non-wasteful* and that *eliminates justified envy* and *respects improvements*.

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- **Bottomline:** In an environment where priorities have to be fully enforced (eg. when priorities are obtained through exams as in Turkey), SOSM is the unambiguous winner!

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- **Bottomline:** In an environment where priorities have to be fully enforced (eg. when priorities are obtained through exams as in Turkey), SOSM is the unambiguous winner!
- **However there is one bit of bad news...**

# Efficiency Cost of Fully Enforcing Priorities

**Example:** There are three students  $i_1, i_2, i_3$  and three schools  $c_1, c_2, c_3$ , each of which has only one seat. Preferences and school priorities are as follows:

$$R_{i_1} : c_2 - c_1 - c_3$$

$$\pi_{c_1} : i_1 - i_3 - i_2$$

$$R_{i_2} : c_1 - c_2 - c_3$$

$$\pi_{c_2} : i_2 - i_1 - i_3$$

$$R_{i_3} : c_1 - c_2 - c_3$$

$$\pi_{c_3} : i_2 - i_1 - i_3$$

Only  $\mu$  eliminates justified envy but it is Pareto dominated by  $\nu$ :

$$\mu = \begin{pmatrix} i_1 & i_2 & i_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$$

$$\nu = \begin{pmatrix} i_1 & i_2 & i_3 \\ c_2 & c_1 & c_3 \end{pmatrix}$$



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$$\pi_{c_2} : i_2 - i_1 - i_3$$

$$R_{i_3} : c_1 - c_2 - c_3$$

$$\pi_{c_3} : i_2 - i_1 - i_3$$

Only  $\mu$  eliminates justified envy but it is Pareto dominated by  $\nu$ :

$$\mu = \begin{pmatrix} i_1 & i_2 & i_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$$

$$\nu = \begin{pmatrix} i_1 & i_2 & i_3 \\ c_2 & c_1 & c_3 \end{pmatrix}$$

- While SOSM Pareto dominates any mechanism that eliminates justified envy, SOSM itself is not Pareto efficient!

# School Choice

- The tension between the full enforcement of priorities and Pareto efficiency might be viewed as an inconvenience with no resolution when priorities are “earned” via exams.

However this is not how priorities are always obtained.

- School choice, first formulated as a market design problem in Abdulkadiroğlu & Sönmez (*AER* 2003), is similar to Turkish student placement problem but it differs in one aspect.

The construction of priority lists is more flexible in school choice, and it can depend on many factors such as home address, sibling status, random tie-breaker, etc.

- This difference might mean that, perhaps it is not the end of the world if one has to be “more flexible” with the enforcement of priority lists. Indeed, this is quite common in the U.S.

## Boston Mechanism

The most widely used mechanism in the U.S. is the mechanism used by Boston Public Schools (BPS) in the period 1988-2005:

1. For each school a priority list is exogenously determined.  
In case of BPS, priority of student  $i$  at a given school  $s$  depends on
  - whether student  $i$  lives in the **walk-zone** of school  $s$ , ,
  - whether student  $i$  has a **sibling** already attending school  $s$ , and
  - a lottery number to break ties.
2. Each student submits a preference ranking of the schools.
3. The final phase is the student assignment based on preferences and priority lists:

# Boston Mechanism

- **Round 1:** In the first round only the first choices of the students are considered. For each school  $s$ , consider the students who have listed  $s$  as first choice and assign seats of school  $s$  to them one at a time following their priority order until either there are no seats left or there is no student left who has listed it as her first choice.
- **Round  $k$ :** Consider the remaining students. In Round  $k$  only the  $k^{\text{th}}$  choices of these students are considered. For each school with still available seats, consider the students who have listed it as their  $k^{\text{th}}$  choice and assign the remaining seats to these students one at a time following their priority order until either there are no seats left or there is no student left who has listed it as her  $k^{\text{th}}$  choice.

## Very Easy to Manipulate

- **Major handicap:** The Boston mechanism is not **strategy-proof**.
- Even if a student has very high priority at school  $s$ , she loses her priority to students who have top ranked school  $s$  unless she lists it as her top choice!
- Hence the Boston mechanism gives parents strong incentives to overrank schools where they have high priority.

## So Easy to Manipulate, it is All Over the News!

Consider the following quotation from St.Petersburg Times:

*Make a realistic, informed selection on the school you list as your first choice. It's the cleanest shot you will get at a school, but if you aim too high you might miss.*

*Here's why: If the random computer selection rejects your first choice, your chances of getting your second choice school are greatly diminished. That's because you then fall in line behind everyone who wanted your second choice school as their first choice. You can fall even farther back in line as you get bumped down to your third, fourth and fifth choices.*

## Evidence from Education Literature

Glenn (PI 1991) states

*As an example of how school selections change, analysis of first-place preferences in Boston for sixth-grade enrollment in 1989 (the first year of controlled choice in Boston) and 1990 shows that the number of relatively popular schools doubled in only the second year of controlled choice. The strong lead of few schools was reduced as others “tried harder.”*

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Highly **optimistic** scenario!

More plausible scenario: **Learning**

## Evidence from 2004-2005 BPS School Guide

- *For a better chance of your “first choice” school . . . consider choosing less popular schools.*

## Lack of Elimination of Justified Envy

- Boston mechanism does not eliminate justified envy: Priorities are lost unless school ranked as top choice.
- Balinski & Sönmez (*JET* 1999): If *elimination of justified envy* is to be maintained, then SOSM is the big winner!
- **Caution:** Recall the efficiency cost of elimination of justified envy.

# Equilibria of the Boston Mechanism

- **Theorem** (Ergin & Sönmez *JPubE* 2006): The set of Nash equilibrium outcomes of the preference revelation game induced by the Boston mechanism is equal to the set of stable matchings of the associated college admissions game under the true preferences.

# Efficiency of the Boston Mechanism

- **Corollary:** The dominant-strategy equilibrium outcome of the SOSM either Pareto dominates or equal to the Nash equilibrium outcomes of the Boston mechanism.
- **Remark 1:** The preference revelation game induced by the Boston mechanism is a huge “coordination game” with lots of uncertainty. So it is unrealistic to expect a Nash equilibrium in practice.  
That said, it is perhaps even less natural to expect an off-equilibrium outcome that is “better” than its best equilibrium.
- **Remark 2:** Ergin & Sönmez (*JPubE* 2006) show that the above result no longer holds under incomplete information.

## Efficiency of the Boston Mechanism

- Abdulkadiroğlu, Che & Yasuda (*AER* 2011) further show that, if
  1. all students have identical ordinal preferences but different cardinal preferences, and
  2. all students have the same claims for each school (and thus priorities at each school is constructed with a uniform lottery)

then any student weakly prefers any symmetric Bayesian Nash equilibria of the Boston mechanism to the dominant strategy outcome of the SOSM.

- Observe that under the above assumptions not only the resulting model is quite different than school choice, but also SOSM is identical to pure random allocation.

# Interim Summary for the Boston Mechanism

- Highly vulnerable to manipulation.
- Does not fully enforce priorities (in the sense of elimination of justified envy).
- Efficiency comparison with SOSM is less clear, but only because the analysis of the preference game induced by the Boston mechanism relies on strong assumptions.

# Doing Away with Elimination of Justified Envy: TTC

We can adopt the **Top Trading Cycles** mechanism (TTC) to school choice:

## Step 1:

- Assign a **counter** for each school which keeps track of how many seats are still available at the school. Initially set the counters equal to the capacities of the schools.
- Each student “points to” her favorite school. Each school points to the student who has the highest priority.
- There is at least one cycle. Every student in a cycle is assigned a seat at the school she points to and is removed. The counter of each school in a cycle is reduced by one and if it reduces to zero, the school is also removed. Counters of all other schools stay put.

**Step k:** Repeat Step 1 for the remaining “economy.”



# Efficiency & Strategy-Proofness of TTC

- TTC simply **trades priorities** of students among themselves starting with the students with highest priorities.
- TTC inherits the plausible properties of Gale's TTC:

**Theorem:** The TTC mechanism is *Pareto efficient* and *strategy-proof*.

## Adoption of SOSM in NYC

- Shortly after Abdulkadiroğlu & Sönmez (*AER* 2003) was published in June 2003, NYC and Boston both adopted the SOSM. However the two reforms evolved in very different ways, and they are summarized in
  - Abdulkadiroğlu, Pathak & Roth (*AER P&P* 2005) for NYC, and
  - Abdulkadiroğlu, Pathak, Roth & Sönmez (*AER P&P* 2005) for Boston.
- **May 2003:** NYCDOE Director of Strategic Planning contacted Alvin Roth for advice on the design of a new high school matching mechanism after the **collapse** of their mechanism.
  - Unlike most other school districts, NYCDOE did not have a direct mechanism prior to 2003.
  - Their mechanism gave students incentives to manipulate their preferences (reminiscent of those under the Boston mechanism), and it gave schools the ability to manipulate their priority ranking as well as to conceal capacity.
  - NYCDOE failed to assign roughly 30 percent of students via its mechanism in its final run, a very visible failure that required abandoning it in haste.

## Adoption of SOSM in NYC

- **October 2003:** NYCDOE adopted SOSM for high school admissions. Strategy-profness of the SOSM made it particularly attractive.

*"For more than a generation, parents and students have been unhappy with the admissions process to New York City high schools. The new process is a vast improvement, as it provides greater choice, equity and efficiency. For example, for the first time, students will be able to list preferences as true preferences, limiting the need to game the system.*

*This means that students will be able to rank schools without the risk that naming a competitive school as their first choice will adversely affect their ability to get into a school they rank lower."*

*Peter Kerr, Director of Communications, NYCDOE*

# Adoption of SOSM in Boston

- Unlike in NYCDOE, BPS was quite satisfied with its mechanism.
- **September 2003:** The *Boston Globe* published an article on Abdulkadiroğlu & Sönmez (*AER* 2003), describing the flaws of the Boston mechanism, and advocating the adoption of SOSM.
- **October 2003:**
  - Following a series of e-mail exchanges, Valerie Edwards, then Strategic Planning Manager at BPS, invited Sönmez to Boston to present the case against the Boston mechanism.

Together with Abdulkadiroğlu, Pathak and Roth, he presented to BPS the case against the Boston mechanism, and proposed two strategy-proof alternatives.
  - While skeptical prior to meeting, BPS staff was convinced strategizing was likely occurring, to the detriment of students and families.
  - They invited the team to carry out an empirical study of the Boston mechanism to support the results in Abdulkadiroğlu & Sönmez (*AER* 2003) and the working paper versions of Chen & Sönmez (*JET* 2006) and Ergin & Sönmez (*JPubE* 2006) presented in the meeting.

## Adoption of SOSM in Boston

- **September 2004:** In their report to BPS, Student Assignment Task Force recommended the adoption of **TTC**.
- **July 2005:** BPS gave up the Boston mechanism and adopted **SOSM**.

**Lesson Learned in the Process:** Over emphasizing certain features of a mechanism via its name, in this case **trade**, can scare off the policy makers!

Policy makers at BPS were mostly worried about the trade of sibling priorities.

# Leveling the Playing Field

## (a.k.a. Getting Rid of the Boston Mechanism)

- Follow up research revealed another major weakness of the Boston mechanism: It favors sophisticated students over naive (or uninformed) students (Pathak & Sönmez *AER* 2008).
- This has been one of the major factors in Boston's decision to abandon the Boston mechanism and adopt the SOSM.

Superintendent Payzant's May 2005 Report to School Committee:

"A strategy-proof algorithm **levels the playing field** by diminishing the harm done to parents who do not strategize or do not strategize well."

- Indeed the Boston mechanism is banned throughout England in 2007 due to similar concerns!

All local authorities in England adopted versions of SOSM in the period 2003-2007 (Pathak & Sönmez *AER* 2013).

## Admissions Reform throughout England

- Aside from Boston (which used the Boston mechanism until 2005), variants of this mechanism have been used in several U.S. school districts including: Cambridge MA, Charlotte-Mecklenburg NC, Denver CO, Miami-Dade FL, Minneapolis MN, Providence RI, and Tampa-St. Petersburg FL.
- U.S. is not the only country where versions of the Boston mechanism are used to assign students to public schools.
- A large number of English Local Authorities had been using what they referred to as “first preference first” systems until 2007.

# Admissions Reform throughout England

- Formally, a **first preference first (FPF)** mechanism is a hybrid between the SOSM and the Boston mechanism: Under this mechanism, a school selects to be either a **first preference first school** or an **equal preference school**, and the outcome is determined by the student-proposing deferred acceptance algorithm, where
  - 1) the base priorities for each student are used for each equal preference school, whereas
  - 2) the base priorities of students are adjusted so that
    - any student who ranks school  $s$  as his first choice has higher priority than any student who ranks school  $s$  as his second choice,
    - any student who ranks school  $s$  as his second choice has higher priority than any student who ranks school  $s$  as his third choice, etc.for each first preference first school.
- The Boston mechanism is a special case of this mechanism when all schools are first preference first schools and the SOSM is a special case when all schools are equal preference schools.



## Ban of the FPF Mechanism in 2007

- 2003 School Admissions Code in England requires all local authorities to coordinate public school admissions.
- While a majority of local authorities adopted truncated versions of the SOSM after (or in anticipation) of the 2003 code, more than 60 local authorities adopted the FPF mechanism (including several that adopted the Boston mechanism).
- The FPF mechanism was banned throughout England with the 2007 School Admissions Code along with other mechanisms that use “unfair oversubscription criteria.”

**Section 2.13:** *In setting oversubscription criteria the admission authorities for all maintained schools must not:*

*... give priority to children according to the order of other schools named as preferences by their parents, including 'first preference first' arrangements.*

## Ban of the FPF Mechanism in 2007

- Rationale given by Department for Education and Skills:  
*'first preference first' criterion made the system unnecessarily complex to parents*
- Education Secretary Alan Johnson remarked that the FPF system “forces many parents to play an ‘admissions game’ with their children’s future.”
- Great deal of public discussion throughout England and striking similarities with concerns about the Boston mechanism at BPS.

# Conclusion

- Design of various institutions is one of the most important roles of the government.
- Good use of economic principles and game theory in these designs can result in significant gains for the society!

That is our agenda in **Market Design**.