Matching with Contracts: Comment

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The *Matching with contracts* model (Hatfield and Milgrom 2005) is widely considered to be one of the most important advances of the last two decades in matching theory. This powerful model embeds the classical matching models of Gale and Shapley (1962) and Kelso and Crawford (1982), and it has given impetus to a flurry of theoretical research as well as to new practical applications of market design. Utilizing fixed-point techniques from lattice theory, Hatfield and Milgrom (2005) analyze the set of stable allocations in their rich framework. One of the main messages of their paper is that the set of stable allocations is non-empty under a ***substitutes*** condition. In this comment we note that an additional ***irrelevance of rejected contracts*** **(IRC)** condition is implicitly assumed throughout their analysis, and in the absence of IRC several of their results, including the guaranteed existence of a stable allocation, fail to hold.[3] The IRC condition requires that the removal of rejected contracts do not affect the choice set, and it is formally stated as follows.

Definition 1 *Given a set of contracts X, a choice function $C : 2^X \to 2^X$ satisfies the* ***irrelevance of rejected contracts*** *if*

$$\forall\, Y \subset X, \forall z \in X \setminus Y \qquad z \notin C(Y \cup \{z\}) \implies C(Y) = C(Y \cup \{z\}).$$

The substitutes condition together with the IRC condition assure the existence of a stable allocation.

Theorem 1 *Suppose for every hospital the choice function satisfies the substitutes condition and the IRC condition. Then the set of stable allocations is non-empty.*

The oversight in the paper emanates from an ambiguity in setting the primitives of the model. While hospital choices from sets of contracts are motivated by underlying hospital preferences, the resulting choice functions are treated as if they were the primitives throughout Hatfield and Milgrom (2005). Choice functions derived from underlying strict hospital preferences automatically satisfy the IRC condition, thereby restoring the results. However this easy fix imposes additional structure on hospital choice functions, beyond that implied by the IRC condition, to maintain the transitivity of the underlying preferences: The hospital choice functions need to satisfy a ***strong axiom of revealed preference*** **(SARP)** condition, in addition to the IRC condition. This in turn means that, not only the role of the substitutes condition in the analysis becomes more opaque being superimposed on an underlying choice structure, but also the scope of the results becomes less general since SARP is not otherwise needed in the analysis. In that sense, letting hospital choice functions to be primitives of the model and assuming IRC throughout the analysis might be the preferred approach to correctly interpret the results of Hatfield and Milgrom (2005).

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[3]See Aygün and Sönmez (2012) for a detailed analysis.

Starting with Blair (1988), there are a number of papers in the matching literature where choice functions are the primitives, and the IRC condition is assumed together with the substitutes condition to prove the existence of stable outcomes along with other results. These papers include Alkan (2002), Alkan and Gale (2003), and Fleiner (2003). Our contribution is showing that the IRC condition is essential for Hatfield and Milgrom (2005) as well. The use of choice functions as primitives is not limited to theoretical matching literature, and it is also the case for some recent market design applications such as cadet-branch matching (Sönmez 2011, Sönmez and Switzer 2011).

Some of the analysis in Hatfield and Milgrom (2005) assumes a ***law of aggregate demand* (LAD)** condition in addition to the substitutes condition. These two conditions together imply the IRC condition and hence our observation has no bite for the results that assume the LAD condition.

References

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