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# Bidding for Army Career Specialties: Improving the ROTC Branching Mechanism

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Motivated by the low retention rates of US Military Academy and Reserve Officer Training Corps graduates, the Army recently introduced incentives programs in which cadets could bid 3 years of additional service obligation to obtain higher priority for their desired branches. The full potential of this incentives program is not utilized because of the ROTC's deficient matching mechanism. I propose a design that eliminates these shortcomings and mitigates several policy problems the Army has identified. In contrast to the ROTC mechanism, my design utilizes market principles more extensively, and it is a hybrid between a market mechanism and a priority-based allocation mechanism.

## I. Introduction

In the last decade there has been a lot of activity and excitement among economists working on matching markets, a field that dates back to the seminal contribution of Gale and Shapley (1962). Theory matured to a point where matching theorists could make policy suggestions in key areas including education and health care.<sup>1</sup> Matching, as a field, owes its recent

I thank John Hatfield, Onur Kesten, Scott Kominers, three anonymous referees, and seminar participants at Berkeley, Boston College, Brown, Chicago, Deakin, Michigan, University of New South Wales, the 2011 National Bureau of Economic Research Market Design Workshop, the 2012 Econometric Society Australasian Meeting, GAMES 2012, Match-Up 2012, the 2012 conference of the Society for the Advancement of Economic Theory, and the 2012 National Science Foundation/Conference on Econometrics and Mathematical Economics Decentralization Conference for helpful comments.

<sup>1</sup> In his recent congressional testimony, Myron Gutmann (assistant director, Social, Behavioral, and Economic Sciences, National Science Foundation) emphasizes that research on matching markets has resulted in measurable gains for the US taxpayer. The testimony is available at [http://www.nsf.gov/about/congress/112/mg\\_sberesearch\\_110602.jsp](http://www.nsf.gov/about/congress/112/mg_sberesearch_110602.jsp).

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success to the discovery of important practical applications backed by solid theory. In this paper, along with a companion paper (Sönmez and Switzer, forthcoming), I introduce a brand new practical application of matching markets: *cadet-branch matching* at US Army programs. While I focus on the evaluation and improvement of cadet branching in the Army, the ideas I develop are substantially more general and can be used in any application in which part of the allocation is based on priorities while markets take over for the rest. Some examples include allocation of school seats with price discrimination based on student talent, on-campus parking, and queues with express lanes.

The two main programs the US Army relies on to recruit officers are the United States Military Academy (USMA) and the Reserve Officer Training Corps (ROTC). Graduates of the USMA and ROTC enter active duty for a period of obligatory service upon completing their programs. The active-duty service obligation (ADSO) is 5 years for USMA graduates, 4 years for ROTC scholarship graduates, and 3 years for ROTC nonscholarship graduates. Upon completion of ADSO, an officer may apply for voluntary separation or continue on active duty. The Army has a strong preference for its officers to serve beyond their ADSO, and the low retention rate of these company-grade officers has been a major issue for the Army since the late 1980s. In the last few years, the Army has responded to this challenge with unprecedented retention incentives, including *branch-for-service* incentives programs offered by both the USMA and the ROTC (Wardynski, Lyle, and Colarusso 2010).

During the fall semester of their senior year, USMA and ROTC cadets “compete” for their branch choices. The outcome of this branching process is essential for cadets not only because it determines their future specialties in the Army but also because career advancement possibilities vary widely across different branches. There has been a long tradition of assigning branches to cadets on the basis of their preferences and their merit ranking. This merit ranking is known as the *order-of-merit list* (OML) in the military and is based on a weighted average of academic performance, physical fitness test scores, and military performance. Until 2006, cadet branching was an application of possibly one of the most straightforward resource allocation problems, and it was solved with a simple mechanism: the top OML cadet was assigned his first choice, the next cadet was assigned his top choice among remaining slots, and so on. This natural mechanism is known as *simple serial dictatorship* and was the mechanism of choice at both the USMA and the ROTC until 2006. Both programs changed their cadet-branching mechanisms that year in response to historically low retention rates of their graduates.

The idea behind this reform is simple: Since branch choice is essential for most cadets, why not allow them to bid an additional period of ADSO for their desired branches? As part of the Army’s broader incentives pro-

gram to combat the high attrition rate, cadets could “buy” increased access at a fraction of slots by agreeing to serve an additional 3 years of active service. The fraction of slots up for bidding is 25 percent for USMA and 50 percent for ROTC. The new matching process is referred to as the *branch-for-service* program for both USMA and ROTC, although the specifics of their two mechanisms are very different. Sönmez and Switzer (forthcoming) show that while the USMA mechanism has several shortcomings, a relatively easy fix consistent with the main features of the USMA design is available. In this paper, I show that the situation is rather different for ROTC, and a more substantial intervention is needed to design a fully satisfactory mechanism. I propose a design that relies more heavily on market principles and show that it not only eliminates the shortcomings of the ROTC mechanism but also mitigates several policy problems the Army has identified.

At first sight the shortcomings of the ROTC mechanism and the USMA mechanism appear to be very similar. They can both yield unfair outcomes in which higher-OML cadets can be envious of their lower-OML peers even when they are willing to pay the increased cost; both mechanisms are vulnerable to preference manipulation, making the branch selection a high-stakes game for cadets; and both mechanisms can encourage cadets to sabotage their OML standing. Indeed, there is evidence of such “tanking” behavior on military Internet forums. What makes the ROTC mechanism a challenge to fix is a skill-based *affirmative action constraint* and the direct method ROTC has chosen to address it. ROTC leaders want to avoid a situation in which cadets of high skill are all concentrated in a few popular branches. To reach that objective they block the access of cadets in the upper half of the OML to the last 35 percent of slots at each branch. Since only the last 50 percent of slots are available for the additional bid, this means that cadets at the upper half of the OML are, to a large extent, excluded from the branch-for-service program. For each of the seven to eight popular branches, this policy resulted in what is referred to as a *dead zone* in the Army jargon, to the severe detriment of a large fraction of cadets.<sup>2</sup> For example, the Aviation dead zone affected cadets in 20–50 OML percentiles in 2011, blocking their access to Aviation slots whether they were willing to pay the additional cost or not. In contrast, their peers in 50–70 OML percentiles had access to all branches, provided that they were willing to pay the extra cost. And not surprisingly, the closer the cadet was to the 50th percentile mark, the more compromised he was. For example, while cadets in 20–30 OML percentiles suffered from the Aviation dead zone alone in 2011, their

<sup>2</sup> The official description of dead zone is given as follows in the 2011 ROTC Accessions Briefing: “Dead Zone: The area on the branch bar graph where it is impossible for a cadet to receive a certain branch.” This document can be obtained from the following link: [http://www.purdue.edu/armyrotc/\\_f/currentcadets/accessions\\_briefing\\_11\\_apr\\_2011.pptx](http://www.purdue.edu/armyrotc/_f/currentcadets/accessions_briefing_11_apr_2011.pptx).

peers in 40–50 OML percentiles suffered from six dead zones (see fig. 1, in part borrowed from an April 11, 2011, ROTC Accessions Process presentation).<sup>3</sup>

It is important to emphasize that it is possible to maintain the above-mentioned discontinuous ROTC priority structure and still fix the vulnerability of the ROTC mechanism to preference manipulation. Nevertheless, the current ROTC priority structure is not compatible with the design of a fully satisfactory mechanism since it relies on the above-mentioned dead zones. So the key question is whether it is possible to implement the Army's distributional goals without creating dead zones. I argue that the answer to this critical question is affirmative although the solution requires the Army to embrace market principles more extensively. Here is my proposal: Cadets can now make a one-time bid of 3 years for the last 50 percent of the slots at each branch. If they were allowed to bid more than 3 years, the role of the OML would decrease and the role of willingness to serve would increase in branch assignment. The idea is that if the highest possible bid is sufficiently high, motivated cadets in the lower half of the OML can outbid their less motivated peers in the upper half of the OML. Similarly, increasing the fraction of slots up for bidding would decrease the role of the OML and increase the role of willingness to serve in branch assignment. The target of assigning at least 35 percent of slots in each branch to cadets in the lower half of the OML can be achieved by a mix of these two adjustments. I refer to the resulting branch priority structure as *bid-for-your-career* (BfYC) priorities.

Finding an "indirect" way to implement the Army's distributional goals is the most challenging part of the design. Once this hurdle is cleared, recent advances in theoretical matching literature by Hatfield and Milgrom (2005) and Hatfield and Kojima (2008, 2010) give us a lot of mileage to design a mechanism that eliminates the above-mentioned shortcomings of the ROTC mechanism: A *cadet-optimal stable mechanism* (COSM) is well defined under BfYC priorities, it is stable, and it Pareto dominates any other stable mechanism. This mechanism always yields a fair outcome in the sense that a high-OML cadet never envies the assignment of his lower-OML peer. Truth-telling is always optimal under COSM (i.e., it is strategy-proof), and an increase in OML standing can only benefit cadets aligning their incentives with the hard work necessary for their academic, physical fitness, and military studies. For the basic case in which no slot is up for bidding, this mechanism reduces to the *Gale-Shapley agent-optimal stable mechanism*,<sup>4</sup> recently adopted by Boston Public Schools for assignment of kindergarten through grade 12 students to public schools, by the

<sup>3</sup> Available at <http://www.purdue.edu/armyrotc/currentcadets/currentcadets.php> under the Accessions Briefing heading.

<sup>4</sup> For the case of a uniform priority list across all branches/schools, the mechanism further reduces to simple serial dictatorship.

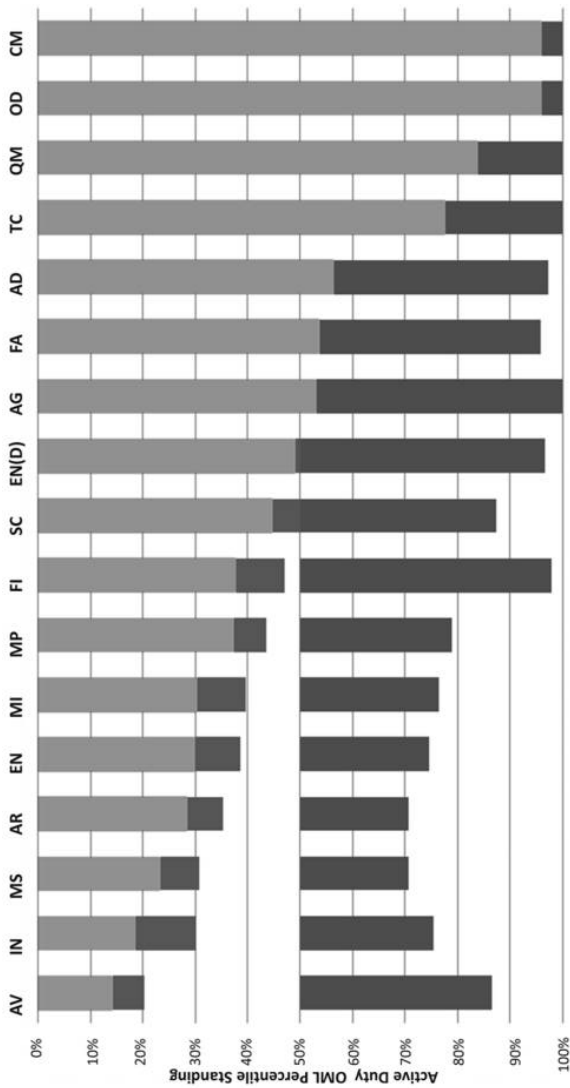


FIG. 1.—ROITC branch assignment results for the year 2011. The vertical axis denotes the active-duty OML percentile standing of the cadets, where 0 percent represents the top cadet and 100 percent represents the last cadet qualified for active duty. Each column represents a branch except for Engineering (EN), which is divided into two parts and represented by two columns. For each branch, the light gray bar represents the part of the OML where cadets secured a slot from the top 50 percent of the slots at the base cost, the darker gray bar above the 50 percentile OML standing represents the part of the OML where cadets secured a slot between 50 and 65 percentiles of the slots at increased cost, and the darkest gray bar below the 50 percentile OML standing represents the part of the OML where cadets received a slot from the last 35 percent of the slots at increased cost. The blank region between the two darker gray bars is the dead zone for the following eight most popular branches: Aviation (AV), Infantry (IN), Medical Service (MS), Armor (AR), Engineering (EN), Military Intelligence (MI), Military Police (MP), and Financial Management (FI). The less sought-after branches without a dead zone are Signal Corps (SC), Engineering reserves for holders of engineering degrees (EN(D)), Adjutant General's Corps (AG), Field Artillery (FA), Air Defense Artillery (AD), Transportation Corps (TC), Quartermaster Corps (QM), Ordnance Corps (OD), and Chemical Corps (CM).

New York City Department of Education for assignment of high school students to public high schools, and throughout England for assignment of students to public schools.<sup>5</sup> The desire to replace highly manipulable student assignment mechanisms with their strategy-proof counterparts was one of the key reasons for these reforms.

The ROTC mechanism is highly deficient from a mechanism design perspective, and replacing it with COSM under BfYC priorities has numerous benefits for cadets, as I have presented. This potential reform also benefits the Army on a number of important policy issues, including (1) better utilization of the branch-for-service incentive program, (2) elimination of the risk of cadets intentionally lowering OML, (3) increased ability to interpret and analyze cadet preferences, and (4) increased flexibility to accommodate branch-specific base priorities. I next briefly discuss these policy benefits.

*Design of a more effective branch-for-service incentive program.*—Branch-for-service incentives were adopted as a response to the Army's major attrition problem. Use of a mechanism that singles out cadets from 20–50 OML percentiles may frustrate them, potentially increasing their attrition rate. In contrast, COSM under BfYC priorities favors cadets who are most willing to serve, thus increasing the cost of leaving the Army right after the base ADSO. By allowing cadets to bid more than 3 years, the mechanism is also likely to significantly boost the man-year gains of the mechanism.

*Removal of incentives to manipulate the system via "tanking."*—The current ROTC mechanism highly encourages cadets from 20–50 OML percentiles to reduce their ranking to avoid the dead zones. Discussions at Service Academy forums suggest that many ROTC cadets are aware of this vulnerability, and some of them engage in manipulative behavior by lowering their OML standing below the median. (See Sec. VI.B for some of the discussions in Service Academy forums.) The Army clearly benefits by adopting a mechanism that promotes high effort levels rather than slacking.

*Credible policy-relevant data generation via strategy-proofness.*—Another major benefit of adopting COSM (even if ROTC priorities are maintained) has to do with its strategy-proofness. A recent study by Lim et al. (2009) investigates the cause of highly undesired minority underrepresentation in leadership ranks of the Army. They observe that while 80 percent of generals are from combat arms branches, minorities do not target these key career branches as much as their white peers. They also observe that minorities tend to have a lower OML than their white peers, and thus the ROTC mechanism gives them an incentive to target less competitive branches. Hence they conclude that part of what seems to be a lack of interest on the part of minorities for combat arms branches might be strategic. They are

<sup>5</sup> See Abdulkadiroğlu et al. (2005), Abdulkadiroğlu, Pathak, and Roth (2005), and Pathak and Sönmez (2011) for the details of reforms in Boston, New York City, and England, respectively.

unable to make policy suggestions since adequate remedies would depend on to what extent the lack of minority representation in key career fields is an artifact of preference manipulation. In simple terms, the Army cannot interpret preference data because its mechanism is highly manipulable. This is entirely avoided under COSM and will allow the Army to implement adequate policies to improve diversity in its senior ranks.

*Possible decentralization of base priorities.*—Another benefit of my proposed reform to the Army is the flexibility of the leadership of each branch to determine its own base priority ranking. This flexibility, which is absent from the current ROTC mechanism, is highly desired for some branches such as Military Intelligence (Besuden 2008).

In addition to introducing a new practical market design problem, this paper, along with Sönmez and Switzer (forthcoming), brings a new perspective to a recent debate in two-sided matching theory. The cadet-branch matching problem is a special case of the *matching with contracts* model (Hatfield and Milgrom 2005), although a substitutes condition key to Hatfield and Milgrom's analysis is not satisfied in this context. The matching with contracts model owes much of its early success to the perception that it subsumes and unifies the Gale and Shapley (1962) college admissions model and the Kelso and Crawford (1982) labor market model, among others.<sup>6</sup> In a surprising result, Echenique (2012) has shown that under the substitutes condition, the matching with contracts model can be embedded within the Kelso and Crawford labor market model, thus showing that the two models are isomorphic.<sup>7</sup> As emphasized by Echenique, the substitutes condition is key for this isomorphism. In particular, that paper indicates that a recent theory paper by Hatfield and Kojima (2010) analyzes matching with contracts under weaker conditions, and his embedding does not work under their conditions. Although Hatfield and Kojima do not offer any applications under their weaker *unilateral substitutes* condition, they show that a number of key results on the agent-optimal stable mechanism persist under this condition. Remarkably, although the substitutes condition fails in my framework, the unilateral substitutes condition is satisfied. Thus cadet-branch matching is the first practical application of the full generality of the matching with contracts model.

## II. The Model

Since 2006, both USMA and ROTC cadets are given an option to sign one or more *branch-of-choice* contracts that increase their priorities at branches

<sup>6</sup> Kelso and Crawford (1982) build on the analysis of Crawford and Knoer (1981).

<sup>7</sup> See also Kominers (2012) for an extension of this isomorphism to many-to-many matching.



of their choosing in exchange for 3 additional years of active military service. A branch-of-choice contract does not guarantee that a cadet will receive a slot at a branch, nor does signing one necessarily oblige him for the 3 additional years of service even if the cadet is assigned a slot at the branch. Loosely speaking, ROTC has changed the priorities for the last 50 percent of the slots at each branch and has given priority to cadets who have committed to 3 additional years of active service.<sup>8</sup> A cadet who signs a branch-of-choice contract is obliged to serve the additional 3 years of service only if he receives a slot from the last 50 percent of the capacity of a branch.

Prior to adoption of branch-for-service programs, cadet-branch matching was an application of what is known as the *student placement problem* in the literature (Balinski and Sönmez 1999). Since the adoption of branch-for-service programs, the nature of the cadet-branch matching problem has changed in two important ways. First of all, the outcome of the problem is no longer merely an assignment of branches to cadets but rather an assignment of branches along with the terms of these assignments. And second, willingness to pay a “higher price” started playing a role in determining who has higher claims at a fraction of the slots. I am now ready to formally introduce the problem.

A cadet-branch matching problem consists of

1. a finite set of cadets  $I = \{i_1, i_2, \dots, i_n\}$ ,
2. a finite set of branches  $B = \{b_1, b_2, \dots, b_m\}$ ,
3. a vector of branch capacities  $q = (q_b)_{b \in B}$ ,
4. a set of “terms”  $T = \{t_1, \dots, t_k\}$ ,
5. a list of cadet preferences  $P = (P_i)_{i \in I}$  over  $(B \times T) \cup \{\emptyset\}$ , and
6. a list of base priority rankings  $\pi = (\pi_b)_{b \in B}$ .

Here  $t \in \mathbb{R}_+$  for each  $t \in T$  with  $t_1 < t_2 < \dots < t_k$ , and a cadet who is assigned the pair  $(b, t)$  commits to serving in the military for at least  $t$  years. For any branch  $b$ , the function  $\pi_b : I \rightarrow \{1, \dots, n\}$  represents the *base priority ranking* of cadets for branch  $b$ , and  $\pi_b(i) < \pi_b(j)$  means that cadet  $i$  has higher claims to a slot at branch  $b$  than cadet  $j$ , other things being equal. For some of the analysis, there will be a uniform base priority ranking across all branches. Following the military terminology, I refer to this uniform ranking as the *order-of-merit list* (OML) and denote it by  $\pi^{\text{OML}}$ . Throughout the paper I fix the set of cadets  $I$ , the set of branches  $B$ , the vector of capacities  $q$ , and the set of terms  $T$ . Hence each problem is defined by a preference list along with a base priority list.

Assume that cadet preferences are strict and are such that for any cadet  $i$ , any pair of branches  $b, b'$ , and any pair of terms  $t, t'$ ,

<sup>8</sup> For the case of the USMA, priorities are changed only at the last 25 percent of the slots.

$$(b, t)P_i(b', t) \Leftrightarrow (b, t')P_i(b', t').$$

That is, each cadet has well-defined preferences over branches that are independent of the service obligation.<sup>9</sup> Let  $\succ_i$  denote cadet preferences over branches alone. For any cadet  $i$ , any pair of branches  $b, b'$ , and any term  $t$ , we have

$$b \succ_i b' \Leftrightarrow (b, t)P_i(b', t).$$

Let  $\mathcal{P}$  denote the set of all preferences over  $(B \times T) \cup \{\emptyset\}$  and  $\mathcal{Q}$  denote the set of all preferences over  $B$ .

A *contract*  $x = (i, b, t) \in I \times B \times T$  specifies a cadet  $i$ , a branch  $b$ , and the terms of their match. Let  $X \equiv I \times B \times T$  be the set of all contracts. Given a contract  $x = (i, b, t)$ , let  $x_I = i$  denote the cadet,  $x_B = b$  denote the branch, and  $x_T = t$  denote the terms of the contract  $x$ . An *allocation*  $X' \subset X$  is a set of contracts such that each cadet appears in at most one contract and no branch appears in more contracts than its capacity. Let  $\mathcal{X}$  denote the set of all allocations. Given a cadet  $i \in I$  and an allocation  $X' \subset X$  with  $(i, b, t) \in X'$ , let  $X'(i) = (b, t)$  denote the *assignment* of cadet  $i$  under allocation  $X'$ . If a cadet  $i$  remains unmatched under allocation  $X'$ , then  $X'(i) = \emptyset$ . A cadet  $i$  prefers an allocation  $X'$  to another allocation  $X''$  if and only if he prefers  $X'(i)$  to  $X''(i)$ . I slightly abuse the notation and use  $P_i$  to denote cadet  $i$ 's preferences over allocations as well, since doing so will not cause any confusion.

**DEFINITION.** For a given problem, an allocation  $X'$  is *fair* if for any pair of cadets  $i, j$ ,

$$X'(j)P_iX'(i) \Rightarrow \pi_{i'}(j) < \pi_{i'}(i),$$

where  $X'(j) = (b', t')$ .

That is, a higher-priority cadet can never envy the assignment of a lower-priority cadet under a fair allocation. Note that it is still possible for a higher-priority cadet to envy the branch assigned to a lower-priority cadet under a fair allocation. Consider a high-priority cadet  $i$  with an assignment  $X'(i) = (b, t)$  and a low-priority cadet  $j$  with  $X'(j) = (b', t')$  with  $t' > t$ . While fairness rules out  $X'(j)P_iX'(i)$ , it is still possible that  $b' \succ_i b$ . A low-priority cadet may be able to get a more preferred branch because he is willing to pay a higher price for it.

A *mechanism* is a *strategy space*  $S_i$  for each cadet  $i$  along with an *outcome function*  $\varphi : (S_1 \times S_2 \times \dots \times S_n) \rightarrow \mathcal{X}$  that selects an allocation for each strategy vector  $(s_1, s_2, \dots, s_n) \in (S_1 \times S_2 \times \dots \times S_n)$ . Given a cadet  $i$  and

<sup>9</sup> While this independence assumption is natural in the present context, it is not essential to this analysis and is made for ease of exposition. The description and evaluation of the ROTC mechanism become more convenient under this assumption.

strategy profile  $s \in S$ , let  $s_{-i}$  denote the strategy of all cadets except cadet  $i$ . A mechanism is *fair* if it always selects a fair allocation. A *direct mechanism* is a mechanism in which the strategy space is the set of preferences  $\mathcal{P}$  for each cadet  $i$ . Hence a direct mechanism is simply a function  $\varphi : \mathcal{P}^n \rightarrow X$  that selects an allocation for each preference profile.

The following highly desirable property of a direct mechanism, when it holds, assures that cadets can never benefit from “gaming” their preferences.

DEFINITION. A direct mechanism  $\varphi$  is *strategy-proof* if there exist no cadet  $i$ , preference profile  $P \in \mathcal{P}^n$ , and preference relation  $P'_i \in \mathcal{P}$  such that  $\varphi(P'_i, P_{-i}) P_i \varphi(P)$ .

One of the most important parameters of the cadet-branch matching problem is the list of base priorities. Clearly, a reasonable mechanism would not penalize a cadet because of an improvement of his base priorities. Given two lists of base priority rankings  $\pi^1, \pi^2$ , we will say that  $\pi^1$  is an *unambiguous improvement* for cadet  $i$  over  $\pi^2$  if

1. the standing of cadet  $i$  is at least as good under  $\pi_b^1$  as  $\pi_b^2$  for any branch  $b$ ,
2. the standing of cadet  $i$  is strictly better under  $\pi_b^1$  than under  $\pi_b^2$  for some branch  $b$ , and
3. the relative ranking of other cadets remain the same between  $\pi_b^1$  and  $\pi_b^2$  for any branch  $b$ .

DEFINITION. A mechanism *respects improvements* if a cadet never receives a strictly worse assignment as a result of an unambiguous improvement in his base priorities.

Observe that the failure of this property hurts the mechanism not only from a normative perspective but also via the adverse incentives it creates in case cadet effort plays any role in determining the base priorities. As in most merit-based resource allocation problems, this is the case for both USMA and ROTC cadet branching.

### III. ROTC Cadet-Branch Matching

Both USMA and ROTC have branch-for-service incentives programs in which cadets can sign branch-of-choice contracts for one or more branches extending their ADSO for 3 years in exchange for increased priority at these branches. Sönmez and Switzer (forthcoming) analyze the USMA mechanism and show that while it has several deficiencies, these can be overcome by a mechanism that is remarkably similar to the USMA mechanism. The situation is somewhat more involved for the ROTC mechanism, and an affirmative action constraint for lower-performance cadets makes the design of a fully satisfactory mechanism a challenge.

### A. ROTC Mechanism

Since 2006,  $T = \{t_1, t_2\}$  for ROTC branching. As such, I refer to  $t_1$  as the *base cost*,  $t_2$  as the *increased cost*, and any contract with increased costs  $t_2$  as a *branch-of-choice contract* in the context of the ROTC mechanism. All cadets are ranked by a single OML denoted by  $\pi^{\text{OML}}$ . Base priority ranking is equal to OML for any branch. That is,  $\pi_b = \pi^{\text{OML}}$  for any branch  $b$ . I am ready to describe the ROTC mechanism.

*Strategy space of the ROTC mechanism:* Each cadet submits a ranking of branches  $\succ'_i$ , and he can sign a branch-of-choice contract for any of his top three choices under  $\succ'_i$ . Let  $B_i$  denote the (possibly empty) set of branches for which cadet  $i$  signs a branch-of-choice contract.

*Outcome function of the ROTC mechanism:* For a given OML and strategy profile  $(\succ'_i, B_i)_{i \in I}$ , the outcome of the ROTC mechanism is obtained as follows: Consider each cadet one at a time, following his OML standing. The treatment of cadets at the top 50 percent of the OML is slightly different from that of those at the bottom 50 percent.

- For each cadet  $i$  at the top half of the OML, consider the following six options in the given order determining his assignment based on the first option he is qualified for. If cadet  $i$  is not qualified for any of the options, then he is left unassigned:
  1. first choice under  $\succ'_i$  at base cost  $t_1$  if less than 50 percent of its slots are full;
  2. first choice under  $\succ'_i$  at increased cost  $t_2$  if less than 65 percent of its slots are full and cadet  $i$  signed a branch-of-choice contract for his first choice;
  3. second choice under  $\succ'_i$  at base cost  $t_1$  if less than 50 percent of its slots are full;
  4. second choice under  $\succ'_i$  at increased cost  $t_2$  if less than 65 percent of its slots are full and cadet  $i$  signed a branch-of-choice contract for his second choice;
  5. third choice under  $\succ'_i$  at base cost  $t_1$  if less than 50 percent of its slots are full;
  6. third choice under  $\succ'_i$  at increased cost  $t_2$  if less than 65 percent of its slots are full and cadet  $i$  signed a branch-of-choice contract for his third choice.
- For cadets at the bottom half of the OML, the options are similar to those of their peers at the top half, except the 65 percent constraint is removed and thus replaced by 100 percent in options 2, 4, and 6.

If a cadet remains unassigned under the ROTC mechanism, his branch assignment is determined by the Department of the Army Branching Board.

*B. Deficiencies of the ROTC Mechanism*

Branch allocation policy at ROTC is in conflict with the design of a fully satisfactory mechanism. For each branch  $b$  the base priority ranking is equal to the OML ranking, and the *ROTC branch priorities* are given as follows:<sup>10</sup>

- For the first 50 percent of the slots, the priority is based on cadet OML ranking.
- The next 15 percent of the slots are reserved for cadets who signed a branch-of-choice contract for branch  $b$ , and among them priority is based on cadet OML ranking.
- The last 35 percent of the slots are reserved for cadets who are at the bottom 50 percent of the OML who signed a branch-of-choice contract for branch  $b$ . Among them priority is based on cadet OML ranking.

Observe that there is an “affirmative action” constraint for the last 35 percent of the slots at each branch, and cadets at the upper half of the OML are denied access to these slots whether they are willing to pay the increased cost or not. As such, the ROTC mechanism fails to be fair. For a given branch, the range of the OML where higher-ranking cadets lose priority to cadets in the lower half of the OML is referred to as the *dead zone* by the Army. In 2011, eight of the most popular branches had a dead zone (see fig. 1). Of those branches, those with the most visible dead zones were Aviation, with a dead zone covering 20–50 OML percentiles, and Infantry, with a dead zone covering 30–50 OML percentiles. In contrast to unfortunate cadets “falling in” one or more dead zones, cadets in the 50–70 OML percentiles had full access to each branch provided that they signed a branch-of-choice contract. The presence of dead zones means that the ROTC mechanism fails to respect improvements, which in turn creates clear incentives to manipulate it via effort reduction. Judging from discussions at military Internet forums, such “tanking” behavior seems present among ROTC cadets. (See Sec. VI.B and App. B for some of these forum posts.)

The ROTC mechanism is vulnerable not only to manipulation via effort reduction but also to preference manipulation. The most obvious reason for this vulnerability is that the ROTC mechanism considers only the top three branch choices, and given the high stakes, cadets need to

<sup>10</sup> Since 2010, the treatment for Engineering is slightly different from that for other branches: half of the slots for Engineering are reserved for cadets with engineering degrees, but otherwise the same priority structure is followed.

choose these branches wisely.<sup>11</sup> The choice of branch-of-choice contracts is not an easy task either, since a branch at increased cost is considered immediately after its cheaper version at the base cost and, more importantly, before any consideration of the lower-ranked branches at the cheaper base cost.<sup>12</sup> In Section IV.C, I discuss in detail why the vulnerability of the ROTC mechanism to preference manipulation is a major obstacle for the US Army in its analysis of a highly debated policy issue.

In order to propose an alternative mechanism that corrects each of these shortcomings, I will relate ROTC cadet-branch matching to an important recent model: matching with contracts. This will also allow me to introduce a third mechanism, which maintains ROTC branch priorities but provides only a partial fix to the shortcomings of the ROTC mechanism.

#### IV. Matching with Contracts

The cadet-branch matching problem is a special case of the matching with contracts model (Hatfield and Milgrom 2005). In the original Hatfield-Milgrom model, each branch (hospitals in their framework) has preferences over sets of agent-cost pairs. These hospital preferences induce a *choice set* from each set of contracts, and it is this choice set (rather than hospital preferences) that is key in the model. In the present framework, branches are not agents and they do not have preferences. However, branches have (potentially elaborate) priorities that also induce choice sets. This is the sense in which the cadet-branch matching problem is a special case of matching with contracts.

I next present the key concepts from matching with contracts to pave the way to propose a new mechanism for ROTC cadet branching. Recall that each cadet  $i$  has preferences  $P_i$  over all branch-cost pairs. Equivalently, each cadet  $i$  has strict preferences over all contracts that include him. A cadet may remain unassigned under the ROTC mechanism. In that case his assignment is determined by the Department of the Army Branching Board (DABB). Hence, in my model, a cadet who is assigned  $\emptyset$  by the ROTC mechanism (or any alternative mechanism) receives a branch that is determined by the DABB. This “lottery” may be more preferred for some cadets than a number of assignments (such as assignments at high cost or assignments with highly undesirable branches). A contract  $(i, b, t)$  is *undesired* for cadet  $i$  if  $\emptyset P_i(b, t)$ . A contract is *acceptable* for a cadet if it is not undesired. Given a set of contracts  $X'$  and a preference relation  $P_i$ , define the *choice of cadet  $i$*  from  $X'$ ,  $C_i(X')$ , to be  $\emptyset$  if all contracts in  $X'$  that include cadet  $i$  are undesired and to be the

<sup>11</sup> The impact of such “truncation” on various matching mechanisms is well analyzed in some related matching problems. See, e.g., Haeringer and Klijn (2009) and Coles and Shorrer (2012).

<sup>12</sup> The USMA mechanism also suffers from this issue (Sönmez and Switzer, forthcoming).

singleton that consists of the most preferred contract of cadet  $i$  in  $X'$  under  $P_i$  otherwise. That is,

$$C_i(X') \equiv \begin{cases} \emptyset & \text{if } \nexists (i, b, t) \in X' \text{ such that } (b, t)P_i \emptyset \\ \{(i, b, t)\} \subseteq X' & \text{if } (b, t)P_i \emptyset \text{ and } (b, t)P_i(b', t') \\ & \text{for any } (i, b', t') \in X' \setminus \{(i, b, t)\}. \end{cases}$$

The *choice of branch  $b$*  from a set of contacts  $X'$ ,  $C_b(X')$ , depends on the policy on who has higher claims for slots at branch  $b$ . These policies are captured by the branch priorities in my model. I refer to  $R_b(X') \equiv X' \setminus C_b(X')$  as the *rejected set*.

A. ROTC Branch Priorities

I next describe the *choice of branch  $b$  under ROTC priorities*: Given a set of contracts  $X'$ , choice of branch  $b$  under ROTC priorities is obtained as follows:

*Phase 0*: Remove all contracts that involve another branch  $b'$  and add them to the rejected set  $R_b^{\text{ROTC}}(X')$ . Hence each contract that survives phase 0 involves branch  $b$ .

*Phase 1.1*: For the first  $0.5q_b$  potential elements of  $C_b^{\text{ROTC}}(X')$ , choose the contracts with highest OML ranking cadets. When two contracts of the same cadet are available, choose the contract with the base cost  $t_1$  and reject the other one, including it in  $R_b^{\text{ROTC}}(X')$ . Continue until either all contracts are considered or  $0.5q_b$  elements are chosen for  $C_b^{\text{ROTC}}(X')$ . If the former happens, terminate the procedure; if the latter happens, proceed with phase 1.2.

*Phase 1.2*: Remove all surviving contracts with base cost  $t_1$  and add them to the rejected set  $R_b^{\text{ROTC}}(X')$ . Proceed with phase 2.1 if there is at least one surviving contract and terminate the procedure otherwise.

*Phase 2.1*: All remaining contracts have increased cost  $t_2$ . Among them include in  $C_b^{\text{ROTC}}(X')$  the contracts with highest OML ranking cadets for the next  $0.15q_b$  potential elements of  $C_b^{\text{ROTC}}(X')$ . Continue until either all contracts are considered or  $0.65q_b$  elements are chosen for  $C_b^{\text{ROTC}}(X')$ . For the former case, terminate the procedure. For the latter case, terminate the procedure if all contracts are considered, and proceed with phase 2.2 otherwise.

*Phase 2.2*: Remove all surviving contracts of cadets from the upper half of the OML adding them to the rejected set  $R_b^{\text{ROTC}}(X')$ . Proceed with phase 3 if there is at least one surviving contract and terminate the procedure otherwise.

*Phase 3*: Each remaining contract is an increased cost contract of a cadet from the lower half of the OML. Among them include in  $C_b^{\text{ROTC}}(X')$

the contracts with highest OML ranking cadets for the last  $0.35q_b$  potential elements of  $C_b^{\text{ROTC}}(X')$ . Reject all remaining contracts and terminate the procedure.

### B. Stability

Since the seminal paper of Gale and Shapley (1962), a condition known as stability has been central to the analysis of two-sided matching markets as well as the allocation of indivisible goods based on priorities.<sup>13</sup> The matching with contracts model has also evolved around the stability condition. Formally, an allocation  $X'$  is *stable* if (1)  $\cup_{i \in I} C_i(X') = X'$ , (2)  $\cup_{b \in B} C_b(X') = X'$ , and (3) there exist no cadet  $i$ , branch  $b$ , and contract  $x = (i, b, t) \in X \setminus X'$  such that

$$\{x\} = C_i(X' \cup \{x\}) \quad \text{and} \quad x \in C_b(X' \cup \{x\}).$$

In the context of cadet-branch matching, the only plausible allocations are the stable ones. Note that if the first requirement fails, then there is a cadet on whom is imposed an undesired contract; if the second requirement fails, then there exists a branch that would rather reject some of its contracts; and if the third requirement fails, then there exists an unselected contract  $(i, b, t)$  in which not only does cadet  $i$  prefer the pair  $(b, t)$  to his assignment, but also contract  $x$  has sufficiently high priority to be selected by branch  $b$ .

### C. Important Properties of Branch Priorities

The following three properties of branch priorities play an important role in the analysis of matching with contracts.

**DEFINITION.** Priorities satisfy the *irrelevance of rejected contracts* (IRC) for branch  $b$  if

$$\forall X' \subset X, \forall x \in X \setminus X', \quad x \notin C_b(X' \cup \{x\}) \Rightarrow C_b(X') = C_b(X' \cup \{x\}).$$

That is, the removal of rejected contracts shall not affect the choice set under the IRC.

**DEFINITION.** Priorities satisfy the *law of aggregate demand* (LAD) for branch  $b$  if

<sup>13</sup> See Roth and Sotomayor (1990) and Sönmez and Ünver (2011) for comprehensive surveys on the role of stability in two-sided matching markets and the allocation of indivisible goods based on priorities.



$$\forall X', X'' \subset X, \quad X' \subset X'' \Rightarrow |C_b(X')| \leq |C_b(X'')|.$$

That is, the size of the choice set never shrinks as the set of contracts grows under the LAD.

DEFINITION. Elements of  $X$  are *substitutes* for branch  $b$  under the choice function  $C_b$  if

$$\forall X', X'' \subset X, \quad X' \subset X'' \Rightarrow R_b(X') \subseteq R_b(X'').$$

That is, contracts are substitutes if any contract that is rejected from a smaller set  $X'$  is also rejected from any larger set  $X''$  that contains  $X'$ . The substitutes condition together with the IRC condition guarantee the existence of a stable allocation (Hatfield and Milgrom 2005).<sup>14</sup>

It is easy to see that ROTC priorities satisfy the IRC condition and the LAD condition, but not the substitutes condition: Consider a cadet  $i$  who is at the lower half of the OML. Between contracts  $x = (i, b, t_1)$  and  $y = (i, b, t_2)$ , the cheap contract  $x$  might be chosen while the expensive one  $y$  is rejected from a smaller set of contracts  $X'$  with lesser competition; although the choice is reversed for  $X'' \supset X'$  where the competition for the slots is higher. A less demanding condition, recently introduced by Hatfield and Kojima (2010), is as follows.

DEFINITION. Elements of  $X$  are *unilateral substitutes* for branch  $b$  if, whenever a contract  $x = (i, b, t)$  is rejected from a smaller set  $X'$  even though  $x$  is the only contract in  $X'$  that includes cadet  $i$ , contract  $x$  is also rejected from a larger set  $X''$  that includes  $X'$ .

I will say that priorities satisfy the unilateral substitutes condition when elements of  $X$  are unilateral substitutes under these priorities.

LEMMA 1. ROTC priorities satisfy the unilateral substitutes condition, the IRC condition, and the LAD condition.

#### D. Cadet-Optimal Stable Mechanism

I am ready to introduce the *cadet-optimal stable mechanism* (COSM), a natural extension of the celebrated agent-optimal stable mechanism (Gale and Shapley 1962): The strategy space of each cadet is  $\mathcal{P}$  under the COSM, and hence it is a direct mechanism. Fix a choice function  $C_b$  for each branch  $b$ . Given a preference profile  $P \in \mathcal{P}$ , the following *cumulative offer algorithm* can be used to find the outcome of the COSM.<sup>15</sup>

<sup>14</sup> While Hatfield and Milgrom (2005) argue that the substitutes condition alone guarantees the existence of a stable allocation, their proof implicitly assumes the IRC condition. See Aygün and Sönmez (2012b) for an example in which the set of stable allocations is empty even though the substitutes condition is satisfied.

<sup>15</sup> The following description is borrowed mostly from Hatfield and Kojima (2010).

*Step 1:* Start the offer process with the highest OML ranking cadet  $\pi^{\text{OML}}(1) = i(1)$ . Cadet  $i(1)$  offers his first-choice contract  $x_1 = (i(1), b(1), t)$  to the branch  $b(1)$  of this contract. Branch  $b(1)$  holds the contract if  $x_1 \in C_{b(1)}(\{x_1\})$  and rejects it otherwise. Let  $A_{b(1)}(1) = \{x_1\}$  and  $A_b(1) = \emptyset$  for all  $b \in B \setminus \{b(1)\}$ .

*Step k:* In general, let  $i(k)$  be the highest OML ranking cadet for whom no contract is currently held by any branch. Cadet  $i(k)$  offers his most preferred previously unrejected contract  $x_k = (i(k), b(k), t)$  to branch  $b(k)$ . Branch  $b(k)$  holds the contract if  $x_k \in C_{b(k)}(A_{b(k)}(k-1) \cup \{x_k\})$  and rejects it otherwise. Let  $A_{b(k)}(k) = A_{b(k)}(k-1) \cup \{x_k\}$  and  $A_b(k) = A_b(k-1)$  for all  $b \in B \setminus \{b(k-1)\}$ .

The algorithm terminates when each cadet either has an offer that is on hold or has no remaining acceptable contracts. Since there is a finite number of contracts, the algorithm terminates after a finite number  $K$  of steps. All contracts held at step  $K$  are finalized resulting in allocation  $\cup_{b \in B} C_b(A_K)$ .

REMARK 1. While the choice of the cadet making the offer at any given step is uniquely defined by the above-described cumulative offer algorithm, the same outcome is obtained regardless of the order of cadets making offers (Hatfield and Kojima 2010).

We will rely on the following pair of results by Hatfield and Kojima (2010).<sup>16</sup>

THEOREM 1. Suppose that the priorities satisfy the unilateral substitutes condition and the IRC condition. Then the cumulative offer algorithm produces a stable allocation. Moreover, this allocation is weakly preferred by any cadet to any stable allocation.

THEOREM 2. Suppose that the priorities satisfy the unilateral substitutes condition, the IRC condition, and the LAD condition. Then the induced COSM is strategy-proof.

### E. A Partial Remedy: COSM under ROTC Priorities

Building on theorems 1 and 2, Sönmez and Switzer (forthcoming) propose *COSM under USMA priorities* to overcome the shortcomings of the USMA mechanism. An important advantage of this approach is that it interferes only with the mechanics of the branch allocation but not with the underlying branch allocation policies. A natural question is whether the same approach eliminates the shortcomings of the ROTC mechanism as well.

<sup>16</sup> Statements of the following two theorems slightly differ from their originals in Hatfield and Kojima (2010), and they include the IRC condition in their hypotheses. The IRC condition is implicitly assumed throughout Hatfield and Kojima's study, and both results fail to hold in its absence (Aygün and Sönmez 2012a).

Let  $\varphi^{\text{ROTC}}$  denote the COSM induced by the ROTC priorities. Before exploring the performance of this mechanism, I will formalize a key failure of the ROTC priorities with strong bearing on the performance of the COSM.

DEFINITION. For any branch  $b$ , branch priorities induced by the choice function  $C_b$  are *fair* if, for any set of contracts  $X'$  and any pair of contracts  $x, y \in X'$  with  $x_B = y_B = b$ ,

$$\begin{aligned} \pi_b(y_I) < \pi_b(x_I), \quad y_T = x_T, \quad \text{and} \quad x \in C_b(X') \\ \Rightarrow \exists z \in C_b(X') \text{ such that } z_I = y_I. \end{aligned}$$

That is, if a contract  $x$  of a lower-priority cadet is chosen, then a contract  $z$  of a higher-priority cadet who is willing to pay as much under a reference contract  $y$  shall also be chosen under fair priorities. Here the chosen contract of cadet  $y_I$  does not need to be the reference contract  $y$ .

OBSERVATION. ROTC priorities are not fair. Cadets from the upper half of the OML are ineligible for the last 35 percent of slots at each branch.

Our first result ties the fairness of the COSM to the fairness of its underlying priorities.

PROPOSITION 1. Suppose that the priorities satisfy the unilateral substitutes condition, the IRC condition, and the LAD condition. Then the COSM is fair if and only if the priorities are fair.

Proposition 1 implies that  $\varphi^{\text{ROTC}}$  cannot fix all shortcomings of the ROTC mechanism. The next result summarizes the strengths and the weaknesses of the COSM under ROTC priorities.

PROPOSITION 2. The outcome of  $\varphi^{\text{ROTC}}$  is stable under ROTC priorities and is weakly preferred by any cadet to any stable allocation. Moreover,  $\varphi^{\text{ROTC}}$  is strategy-proof. However, neither is it fair nor does it respect improvements.

COSM under ROTC priorities is an improvement over the ROTC mechanism in that it is strategy-proof and stable. Nevertheless, it is far from a fully satisfactory mechanism: it fails to be fair and harbors incentives for effort reduction. These observations suggest that it is necessary to seek an alternative priority structure in order to design a fully satisfactory mechanism.

## V. Bidding for Priorities

As I have already argued, current ROTC priorities are not compatible with a fully satisfactory mechanism. This observation leads to the following natural question: Could it be possible to reach the Army’s distributional goals without creating a dead zone? I argue that the answer to this key question is affirmative. My approach is based on increasing the

highest price cadets can bid for their desired branches and adjusting branch priorities in the following way:

- the first  $\lambda$  fraction of the slots will be allocated following the OML, whereas
- cadets who are willing to serve in the military longer will have higher priority for the last  $(1 - \lambda)$  fraction of the slots.

There are currently two possible “prices” for branch assignment under the ROTC mechanism: the base price and the increased price, that is, 3 years in addition to the base price. Under the proposed mechanism, the set of terms is larger, and cadets are able to bid more than 3 years of increased service. In particular, we need the highest price to be sufficiently large, a price only the most motivated cadets will be willing to pay.<sup>17</sup> This will decrease the role of the OML and increase the role of willingness to serve in branch priorities. Another factor that will shift the balance in favor of willingness to serve is increasing the fraction of slots up for bidding. The idea is that the Army’s distributional goals can be met if the role of willingness to serve is sufficiently increased and the role of the OML is sufficiently decreased in branch priorities.<sup>18</sup> Clearly the lower the parameter  $\lambda$  is, the higher the access of lower-OML cadets for highly sought branches, provided that they are willing to pay the price.<sup>19</sup> In contrast to the current mechanism, there will not be arbitrary dead zones and motivation to serve the Army will play a more important role in cadet branching. The proposal builds on the following perspective offered by Wardynski et al. (2010, 63), who played a central role in the design of branch-for-service incentives programs:

The branch and post incentives also raised concerns. Devoted supporters of the ROTC and West Point Order of Merit (OML) system for allocating branches and posts objected that low OML cadets could buy their branch or post of choice ahead of higher OML cadets. Since branch and post assignments represent a zero sum game,

<sup>17</sup> It is important to emphasize that increasing the highest possible bid beyond 3 years is a feasible design consideration. In addition to branch-for-service incentives, the Army has a *post-for-service* incentives program as well as a *graduate school-for-service* incentives program, and cadets are allowed to apply for up to two of these programs for a total of 6 years of additional ADSO.

<sup>18</sup> Implicit here is the assumption that there is no strong positive correlation between the OML and willingness to serve. This is a sensible assumption since Wardynski et al. (2010) report that the retention rate is lower among higher-OML cadets.

<sup>19</sup> Since we take an indirect route to implement the Army’s affirmative action constraint, a particular choice of parameters might not guarantee solutions that satisfy the constraints currently enforced. Hence optimal calibration of the proposed mechanism is an important empirical exercise.

the ability of cadets with a lower OML ranking to displace those above them was viewed by some as unfair or as undermining the OML system. However, rather than undermining the legacy system or creating inequities, the branch and post incentives program makes willingness to serve a measure of merit in branching and posting, thus providing taxpayers a fair return on their officer accessions investment.

For a given  $\lambda$  and set of terms  $T = \{t_1, \dots, t_k\}$ , the choice of branch  $b$  from a set of contracts  $X'$  is obtained as follows under the proposed BfYC priorities.

*Phase 0:* Remove all contracts that involve another branch  $b'$  and add them to the rejected set  $R_b^{\text{BfYC}}(X')$ . Hence each contract that survives phase 0 involves branch  $b$ .

*Phase 1:* For the first  $\lambda q_b$  potential elements of  $C_b^{\text{BfYC}}(X')$ , choose the contracts with highest OML ranking cadets one at a time. When multiple contracts of the same cadet are available, choose the contract with the lowest cost and reject the other ones, including them in  $R_b^{\text{BfYC}}(X')$ . Continue until either all contracts are considered or  $\lambda q_b$  slots are full. If the former happens, terminate the procedure; if the latter happens, proceed with phase 2.

*Phase 2:* For the last  $(1 - \lambda)q_b$  potential elements of  $C_b^{\text{BfYC}}(X')$ , choose the contracts with highest costs while using the OML to break ties. When multiple contracts of the same cadet are available, choose the contract with the highest cost and reject the other ones, including them in  $R_b^{\text{BfYC}}(X')$ . Continue until either all contracts are considered or the capacity is full. Reject any remaining contracts.

REMARK 2. In order to deviate minimally from the current ROTC priorities, I defined BfYC priorities on the basis of a fixed base priority ranking, the OML, for each branch. Clearly BfYC priorities can be based on branch-specific base priority rankings. This flexibility is one of the advantages of the proposed mechanism.

The next lemma implies that BfYC priorities are compatible with the design of a satisfactory mechanism.

LEMMA 2. BfYC priorities satisfy the unilateral substitutes condition, the IRC condition, and the LAD condition, and they are fair.

*A complete remedy: COSM under BfYC priorities.*—Lemma 2 implies that COSM is well defined under the BfYC priorities. Let  $\varphi^{\text{BfYC}}$  denote the COSM induced by BfYC priorities. This mechanism fixes all previously mentioned shortcomings of the ROTC mechanism.

PROPOSITION 3. The outcome of  $\varphi^{\text{BfYC}}$  is stable under BfYC priorities, and it is weakly preferred by any cadet to any stable allocation. Moreover,  $\varphi^{\text{BfYC}}$  is strategy-proof and fair, and it respects improvements.

REMARK 3. COSM under BfYC priorities is a hybrid with features of both a market mechanism and a priority-based allocation mechanism. It takes the form of the Gale and Shapley (1962) agent-optimal stable mechanism for the case of  $\lambda = 1$  and a version of the Kelso and Crawford (1982) labor market algorithm for the case of  $\lambda = 0$ . The latter can be interpreted as a dynamic auction (cf. Gül and Stacchetti 2000), an important feature that is also shared by COSM under BfYC priorities. The relation between this dynamic cadet-branching auction and COSM under BfYC priorities is reminiscent of the relation between the English auction and the second-price sealed-bid auction. The dynamic auction format can be a more convenient alternative if the size of the strategy space of  $\varphi^{\text{BfYC}}$  becomes impractically large.

Indeed, the following more general characterization holds for the COSM.

PROPOSITION 4. Suppose that the priorities satisfy the unilateral substitutes condition, the IRC condition, and the LAD condition. Then COSM is the only mechanism that is stable and strategy-proof.

I can summarize the main findings as follows: (1) Compared to the ROTC mechanism, the Army gains *stability* and *strategy-proofness* by maintaining the original ROTC priorities and adopting the COSM. (2) In addition, the Army gains *fairness* along with *respect for improvements* by replacing ROTC priorities with BfYC priorities.

## VI. Policy Implications for the Army

From a mechanism design perspective, the ROTC mechanism is a severely deficient mechanism. I next argue that this is not only a matter of theoretical aesthetics; the elimination of these shortcomings mitigates several policy problems that the Army has identified.

### A. Better Utilization of Branch-for-Service Incentives Program

Since the 1980s, the US Army has experienced very low retention rates among its most junior officers. This important problem has been well analyzed, and it is estimated that about 75–80 percent of the required officers at the ranks of major and captain are available today (Wardynski et al. 2010). The low retention rates of USMA and ROTC graduates mean that the Army loses much of its ability to screen the quality of its officers for higher ranks. This is evidenced by promotion rates well beyond Defense Officer Personnel Management Act (DOPMA) target rates, as well as the shortened times between promotion opportunities. In contrast to DOPMA target rates of 95 percent and 80 percent, the promotion rates in 2005 to the ranks of captain and major were 98.4 percent and 97.7 percent, respectively (Henning 2006). Similarly, between 1992

and 2004, the share of captains with less than 4 years of active federal commissioned service rose from 8 percent to 30 percent (Wardynski et al. 2010). To make matters worse, the retention rate of higher-quality officers is particularly low, perhaps because they have especially appealing outside opportunities.

The introduction of branch-for-service incentives programs is a response to this problem. However, restricting cadet bids to only a one-time bid of 3 additional years reduces the potential impact of the mechanism. Moreover, cadets between 20 and 50 OML percentiles are to a large extent shut off from the branch-for-service program because of the dead zones. Favoring low-performing cadets at the expense of these moderately well-performing cadets not only undermines the OML system but also potentially aggravates their attrition rate. The adoption of the COSM under BfYC priorities will not only allow cadets to bid more than 3 years for their desired career specialties but also allow the Army to distribute talent across branches on the basis of cadet willingness to serve rather than artificially created dead zones. Instead of favoring arbitrary low-performing cadets, our proposed mechanism favors cadets who are most eager to serve in the Army.

### *B. Elimination of the Risk of Cadets Intentionally Lowering OML*

Since the ROTC mechanism severely penalizes cadets in 20–50 OML percentiles, it gives them strong incentives to reduce their efforts in their studies. This incentive is especially strong for cadets just above the median cadet, since they can avoid losing access to career branches with a relatively small “compromise” in their OML.<sup>20</sup> A mechanism that promotes such behavior can clearly compromise the Army’s efforts in investing in its future. The following post borrowed from the Service Academy forums suggests that ROTC cadets do engage in such manipulative behavior and that the vulnerability of the ROTC mechanism to such gaming of the system is of concern in the military community: “Are you saying they should have tried to get E at LDAC, which would have put them at 46% AD OML, and then NOT gotten their choice of branch? I don’t disagree with you at all, but these kids in the DEAD ZONE are 20 year old cadets being faced with a moral dilemma . . . do your best and

<sup>20</sup> The ROTC mechanism is not the only mechanism that harbors incentives for effort reduction. For most mechanisms, however, expecting agents to materialize such incentives would be unrealistic, for doing so would require masses of information agents cannot have. This is where the ROTC mechanism stands out, and manipulating it through effort reduction is rather easy. Indeed, the Army provides all the necessary data that are needed for a successful manipulation at the following link: [http://www.career-satisfaction.army.mil/pdfs/Order\\_of\\_Merit\\_Score\\_Calculations.pdf](http://www.career-satisfaction.army.mil/pdfs/Order_of_Merit_Score_Calculations.pdf). The data in this document include the OML score for the median cadet for the year 2010 as well as the effect of a 1-point increase in OML on the OML ranking around the median.



kiss your branch choice goodbye, or screw up and get your Branch choice. I think this is a strange choice to put in front of a young cadet." In Appendix B, I give several additional forum posts on the same theme.<sup>21</sup>

In contrast to the ROTC mechanism, the proposed mechanism respects improvements in cadet performance, and thus cadets can only benefit from an improvement of their OML ranking. Simply put, COSM under BfYC priorities fully aligns cadets' interests with those of the Army.

### *C. Branch Choice and Diversity among Senior Military Officers*

While the lower ranks of the US military exhibit a high level of demographic diversity, its higher ranks have remained demographically homogeneous. In 2006, while 31 percent of the enlisted ranks were African American/Hispanic, only about 16 percent of all officers were African American/Hispanic, and only 5 percent of all generals were African American/Hispanic (Lim et al. 2009). This is cause for major concern for the military, and significant resources have been devoted to analyzing this phenomenon. In a recent RAND Corporation report prepared for the Office of the Secretary of Defense, Lim et al. conclude that the relative scarcity of minorities in combat arms branches of the Army is a potential barrier to improving demographic diversity in the senior officer ranks. In 2006, 80 percent of all generals were from combat arms branches. Using 2007 Army ROTC data, Lim et al. show that while 58 percent of white cadets' submitted first choices were in combat arms, only 31 percent of African American cadets' first choices were in combat arms. They also report that minorities tend to rank lower on the OML and conclude that these numbers may not truly reflect a lack of interest on the part of minorities for combat arms. The following quote is from Lim et al.'s report (2009, 25):

In this exploratory study, we have demonstrated that it is critical for the Army to increase minority representation in key career fields to improve the racial and ethnic diversity of its top military officers. But we also contend that there is a strong need for a more in-depth analysis of the Army branching process. If, as our study suggests, minorities are indeed self-selecting into career fields with relatively limited promotion opportunities, why are they doing so? On the one hand, minority cadets could truly prefer different career fields than white cadets. In this case, policy should focus on ways to make combat career fields

<sup>21</sup> Forum posts are available at <http://www.serviceacademyforums.com/showthread.php?t=26435>.



more appealing to minorities. On the other hand, minorities may not really prefer support career fields but rather may reason that they lack the OML to get a more competitive career field (or they forecast a low probability of success in that career field). In this case, minority cadets might desire a Combat Arms career field but may opt for their most-preferred Combat Support or Combat Service Support career field thinking that they would never get a top Combat Arms assignment.

The authors are unable to interpret ROTC preference data because they do not know to what extent minorities strategically avoided more competitive career fields (to avoid a forced assignment). This ambiguity would have been avoided had ROTC used a strategy-proof mechanism. The vulnerability of the ROTC mechanism to preference manipulation thus has adversely affected the authors' ability to prescribe an adequate policy recommendation in this important analysis.<sup>22</sup> There are also several other studies emphasizing the need for understanding minority preferences. The following quote is from Clark (2000, 74–75): “Another area for future research should focus the issue more closely on the branch selection process in commissioning sources. This issue requires a broad quantitative study to determine the predominant factors in branch selection for black officer candidates from ROTC and USMA. Insight into these factors could lead to inventive solutions to increase ethnic diversity in the combat arms.” These and numerous similar studies show that the adoption of a strategy-proof mechanism is highly valuable to ROTC. Hence even if ROTC maintains its current priority structure that relies on dead zones, adoption of COSM will eliminate the difficulties the Army faces in preference data interpretation and allow it to adopt adequate policies to combat minority underrepresentation in its senior ranks.

#### *D. Flexibility to Accommodate Branch-Specific Priorities*

ROTC leadership currently distributes talent across branches by shutting off the upper half of the OML from the last 35 percent of slots at each

<sup>22</sup> The failure to interpret cadet preference data is also acknowledged by the US military leadership. The following quote is taken from Military Leadership Diversity Commission (2010, 4): “The high concentration of white male officers in the flag ranks is partly a result of the high concentration of white male officers in tactical occupations. Recent research and data suggest that differences in initial career field preferences partly explain the high concentration of white male officers in tactical operations. However, little is known about the reasons why initial officer occupational preferences differ along racial/ethnic or gender lines. Regardless of the reasons for this difference, initial officer occupational classification has important implications for demographic diversity at the highest ranks of military leadership.”

branch. This direct approach relies on the use of a uniform base priority ranking across all branches. Leadership at some of the branches has been critical of this practice. The following quote is from Besuden (2008, 1), who argues that the base priorities for Military Intelligence should be improved: "The Army Reserve Officer Training Corps (ROTC) accessions process does not serve either the needs of the Army or the cadets attempting to get one of their top branch choices. The accessions process overvalues certain aspects of a cadet's background and puts no value on other aspects that could indicate a cadet's potential. In order to get and retain those cadets who are best suited to be Military Intelligence Officers, the accessions process must be changed to better reflect the key competencies of an MI Officer." Another reason why many are critical about the use of ROTC-OML as the uniform base priority across all branches has to do with the unusually diverse backgrounds of ROTC cadets. Army ROTC is offered in more than 270 universities and colleges across the United States. Hence the quality of education varies widely across ROTC programs. The weight of academic performance, measured by cadet grade point average in the first 3 years of college, is 40 percent in the OML calculation, and the lack of a standard causes ROTC-OML to be overly subjective.

My proposed mechanism, unlike the ROTC mechanism, is fully flexible on the choice of base priorities.

## VII. Conclusion

Market design owes much of its recent success to the discovery of new practical applications that are supported by elegant theory. Starting with the mid-1990s, multiobject auctions have been employed to allocate radio spectrum, electricity, and timber, involving hundreds of billions of dollars worldwide (Milgrom 2004). More recent applications include student admissions (Balinski and Sönmez 1999; Abdulkadiroğlu and Sönmez 2003; Ergin and Sönmez 2006; Ergin and Erdil 2008; Pathak and Sönmez 2008; Kesten 2010), kidney exchange (Roth, Sönmez, and Ünver 2004, 2005, 2007; Ünver 2010), course allocation (Sönmez and Ünver 2010; Budish 2011; Budish and Cantillon 2012), and Internet ad auctions (Edelman, Ostrovsky, and Schwarz 2007; Varian 2007). In this paper I introduced a new practical application of market design. I present the case for the replacement of the ROTC mechanism and argue that the Army's distributional goals can be implemented through more extensive use of market principles. While the focus has been on a potential reform of the ROTC mechanism, my intention is also the introduction of a resource allocation model in which part of the allocation is based on priorities and market principles take over the rest.

Appendix A

Proofs

*Proof of Lemma 1*

Let  $C_b^{\text{ROTC}}$  be the choice function for branch  $b$  under ROTC priorities. For  $Y \subseteq X$ , let  $C_b^1(Y)$ ,  $C_b^2(Y)$ , and  $C_b^3(Y)$  denote the set of contracts included in  $C_b^{\text{ROTC}}(Y)$  in phase 1, phase 2, and phase 3, respectively.

1. *ROTC priorities satisfy the unilateral substitutes condition:* Let  $X' \subset X$  be a set of contracts and let  $x = (i, b, t) \in X'$  be the only contract of cadet  $i$  in  $X'$ . Suppose  $x \notin C_b^{\text{ROTC}}(X')$ . Clearly  $x \notin C_b^1(X')$ ,  $x \notin C_b^2(X')$ , and  $x \notin C_b^3(X')$ . Let  $X'' \supset X'$ . Since  $x \notin C_b^1(X')$ , there are at least  $0.5q_b$  cadets with contracts in  $X'$  who have higher OML priority than cadet  $x_i$ . Since each of these cadets competes for slots in  $C_b^1(X'')$  as well,  $x \notin C_b^1(X'')$ .

Next without loss of generality (w.l.o.g.) assume  $x_T = t_2$ , for otherwise contract  $x$  does not qualify for slots in  $C_b^2(X'')$  or  $C_b^3(X'')$ . Since the minimum OML priority needed for phase 1 slots is at least as high under  $X''$  as in  $X'$ , any cadet who cannot secure a slot in phase 1 under  $X'$  also fails to receive one under  $X''$ . Hence any contract (with higher cost  $t_2$ ) that is considered for a phase 2 slot under  $X'$  is also considered under  $X''$ , which implies that the minimum OML priority needed for phase 2 slots is at least as high under  $X''$  as in  $X'$ . Thus  $x \notin C_b^2(X')$  implies that there are at least  $0.15q_b$  higher-priority cadets than cadet  $x_i$  with higher-cost contracts in  $X''$  who fail to receive a slot in phase 1 under  $X''$ , which in turn implies  $x \notin C_b^2(X'')$ .

Finally, w.l.o.g. assume that cadet  $x_i$  is in the lower half of the OML priority ranking, for otherwise he does not qualify for slots in  $C_b^3(X'')$ . Since the minimum priorities needed for phase 1 and phase 2 slots are both at least as high under  $X''$  as under  $X'$ , any contract that is considered for a phase 3 slot under  $X'$  is also considered under  $X''$ . Hence the minimum OML priority needed for phase 3 slots is at least as high under  $X''$  as in  $X'$ , and thus  $x \notin C_b^3(X')$  implies  $x \notin C_b^3(X'')$ . Hence  $x \notin C_b^{\text{ROTC}}(X'')$ , and therefore, ROTC priorities satisfy the unilateral substitutes condition.

2. *ROTC priorities satisfy the IRC:* Immediately follows since rejected contracts have no effect on the choice of other contracts under ROTC priorities.

3. *ROTC priorities satisfy the LAD:* Let  $X' \subset X''$ . All contracts are eligible for slots chosen in phase 1. Since every agent with a contract in  $X'$  also has one in  $X''$ , we have  $|C_b^1(X'')| \geq |C_b^1(X')|$ . Moreover,  $X' \subset X''$  implies that the minimum OML priority needed for phase 1 slots is at least as high under  $X''$  as in  $X'$ . That means that a cadet who cannot secure a slot in phase 1 under  $X'$  also fails to receive one under  $X''$ . Hence any contract (with higher cost  $t_2$ ) that is considered for a phase 2 slot under  $X'$  is also considered under  $X''$ . Thus  $|C_b^2(X'')| \geq |C_b^2(X')|$ . It also implies that, as in the case of phase 1 slots, the minimum OML priority needed for phase 2 slots is at least as high under  $X''$  as in  $X'$ . Finally, consider a cadet  $i$  who is in the lower half of the OML and suppose  $(i, b, t_2) \in C_b^3(X')$ . Clearly cadet  $i$  does not meet the priority threshold for phase 1 or phase 2 slots under  $X'$  and thus under  $X''$  as well. Hence contract  $(i, b, t_2)$  is one of the contracts to compete for

phase 3 slots under  $X''$ , and hence  $|C_b^3(X'')| \geq |C_b^3(X')|$ . Since  $C_b^1(X'')$ ,  $C_b^2(X'')$ , and  $C_b^3(X'')$  are disjoint,  $|C_b^s(X'')| \geq |C_b^s(X')$  for  $s = 1, 2, 3$  implies  $|C_b^{\text{ROTC}}(X'')| \geq |C_b^{\text{ROTC}}(X')|$ , showing that ROTC priorities satisfy the LAD. QED

*Proof of Proposition 1*

By theorem 1, COSM is well defined when priorities satisfy the unilateral substitutes condition along with the IRC condition. Let  $\varphi$  denote the COSM for given priorities.

*Priorities are fair  $\Rightarrow$  COSM is fair:* Fix a choice function  $C_b$  for each branch  $b$ . Suppose that the induced COSM is not fair. Then there exists a preference profile  $P$  and a pair of agents  $i, j$  such that

$$\varphi_j(P) P_i \varphi_i(P),$$

where  $\varphi_j(P) = (b, t)$ ,  $\varphi_i(P) = (b', t')$ , and  $\pi_b(i) < \pi_b(j)$ . We can assume w.l.o.g. that contract  $(j, b, t)$  is the only acceptable contract for cadet  $j$  under  $P_j$ , since the choice function for each branch will remain the same under this transformation by IRC, and the mechanics of the cumulative offer algorithm will assure that the same allocation will be obtained by the COSM.

Observe that cadet  $i$  prefers contract  $y = (i, b, t)$  to  $z = (i, b', t')$ . Therefore, contract  $y$  must be offered to branch  $b$  but rejected by the cumulative offer algorithm. Let  $X'$  be the set of contracts on hold by the cumulative offer algorithm when  $y$  was rejected. By choice of  $X'$ , not only  $y \notin C_b(X' \cup \{y\})$ , but also cadet  $i$  does not have an alternative contract in  $X'$  to be considered by the choice function. Let  $x = (j, b, t)$  and define  $X'' = X' \cup \{x, y\}$ . Since priorities satisfy the unilateral substitutes condition,  $y \notin C_b(X' \cup \{y\})$  implies  $y \notin C_b(X'')$ .

Next define  $X'''$  to be the set of contracts offered to branch  $b$  until the cumulative offer algorithm has terminated. We have (1)  $X'' \subset X'''$  and (2)  $x \in C_b(X''')$  since  $\varphi_j(P) = (b, t)$ . Moreover, since  $x$  is the only acceptable contract under  $P_j$ , cadet  $j$  does not have an alternative contract in  $X'''$  to be considered by the choice function. Therefore, again by the unilateral substitutes condition we must have  $x \in C_b(X''')$ . Hence  $x_T = y_T$ ,  $\pi_b(i) < \pi_b(j)$ ,  $x \in C_b(X''')$ , and yet  $y \notin C_b(X''')$ . Finally, by construction, cadet  $i$  has no contract other than  $y$  in  $X'''$ , and thus he has no contract in  $C_b(X''')$ . This shows that priorities are not fair.

*COSM is fair  $\Rightarrow$  priorities are fair:* Suppose that the priorities are not fair. For any branch  $b$ , let  $C_b$  be the choice function induced by the underlying priorities. Then there exist a branch  $b$ , a set of contracts  $X'$ , a pair of contracts  $x, y \in X'$  with  $x_B = y_B = b$ ,

$$\pi_b(y_T) < \pi_b(x_T), \quad y_T = x_T, \quad \text{and} \quad x \in C_b(X') \\ \text{and yet } \nexists z \in C_b(X') \text{ such that } z_T = y_T.$$

Let  $i = y_i$  and  $X'' = C_b(X')$ . Construct the following preference profile  $P$ :

1. For cadet  $i$ , let  $(i, b, y_T) = (i, b, x_T) = y$  be the only acceptable contract under  $P_i$ .

2. For any agent  $j \in X''_i$  with  $x'' = (j, b, x''_T) \in C_b(X')$ , let  $x''$  be the only acceptable contract.
3. For any remaining cadet, let there be no acceptable contract.

By IRC,  $C_b(X'') = C_b(X') = X''$ . Therefore,  $C_b(X'' \cup \{y\}) = X''$  by unilateral substitutes along with LAD, which in turn implies  $\varphi(P) = X''$  since only contracts in  $X'' \cup \{y\}$  are offered under the cumulative offer algorithm. But then

$$\varphi_{x_i}(P)P_i\varphi_i(P),$$

where  $\varphi_{x_i}(P) = (b, x_T) = (b, y_T)$  and  $\varphi_i(P) = \emptyset$ , even though  $\pi_b(i) < \pi_b(x_i)$ . Hence COSM is not fair. QED

*Proof of Lemma 2*

Let  $C_b^{BfYC}$  be the choice function for branch  $b$  under BfYC priorities.

1. *BfYC priorities satisfy the unilateral substitutes condition:* Let  $X' \subset X$  be a set of contracts and let  $x = (i, b, t) \in X'$  be the only contract of cadet  $i$  in  $X'$ . Suppose  $x \notin C_b^{BfYC}(X')$ . Let  $x^1$  be the last contract picked for  $C_b^{BfYC}(X')$  in phase 1 and  $x^2$  be the last contract picked for  $C_b^{BfYC}(X')$  in phase 2 of construction of  $C_b^{BfYC}(X')$ . We have  $\pi_b(x^1_i) < \pi_b(i)$ , for otherwise contract  $x$  would have been picked for  $C_b^{BfYC}(X')$  in phase 1 before contract  $x^1$ . Similarly, we have  $x^2_T \geq t$ , and if  $x^2_T = t$ , then  $\pi_b(x^2_T) < \pi_b(i)$ ; otherwise contract  $x$  would have been picked for  $C_b^{BfYC}(X')$  in phase 2 before contract  $x^2$ .

Let  $X'' \supset X'$ . Let  $y^1$  be the last contract picked for  $C_b^{BfYC}(X'')$  in phase 1 and  $y^2$  be the last contract picked for  $C_b^{BfYC}(X'')$  in phase 2 of construction of  $C_b^{BfYC}(X'')$ . Since  $X'' \supset X'$ , the thresholds to be picked are at least as competitive under  $X''$ , and hence (1)  $\pi_b(y^1_i) \leq \pi_b(x^1_i) < \pi_b(i)$  and (2)  $y^2_T \geq x^2_T \geq t$  and  $y^2_T = x^2_T = t \Rightarrow \pi_b(y^2_T) \leq \pi_b(x^2_T) < \pi_b(i)$ . Therefore, contract  $x$  is not chosen for  $C_b^{BfYC}(X'')$  either in phase 1 or in phase 2 showing  $x \notin C_b^{BfYC}(X'')$ . Hence BfYC priorities satisfy the unilateral substitutes condition.

2. *BfYC priorities satisfy the IRC:* Immediately follows since rejected contracts have no effect on the choice of other contracts under BfYC priorities.

3. *BfYC priorities satisfy the LAD:* By construction of the BfYC chosen set, all contracts of a given cadet can be rejected from a branch only when it reaches full capacity. Hence the size of the BfYC chosen set can never shrink as the set of available contracts grows.

4. *BfYC priorities are fair:* Let the set of contracts  $X' \subseteq X$  and contracts  $x, y \in X'$  with  $x_B = y_B = b$  be such that  $\pi_b(y_i) < \pi_b(x_i)$ ,  $y_T = x_T$ , and  $x \in C_b^{BfYC}(X')$ . Contract  $x$  is picked for  $C_b^{BfYC}(X')$  either in phase 1 or in phase 2. If  $x$  is picked for  $C_b^{BfYC}(X')$  in phase 1, then  $\pi_b(y_i) < \pi_b(x_i)$  implies that the lowest-cost contract of cadet  $y_i$  in  $X'$  is picked for  $C_b^{BfYC}(X')$  in phase 1 before contract  $x$ . Since  $y \in X'$ , such a contract exists. If  $x$  is picked for  $C_b^{BfYC}(X')$  in phase 2,  $\pi_b(y_i) < \pi_b(x_i)$  and  $y_T = x_T$  imply that either the lowest-cost contract of cadet  $y_i$  in  $X'$  is picked for  $C_b^{BfYC}(X')$  in phase 1 or the highest-cost contract of cadet  $y_i$  in  $X'$  is picked for  $C_b^{BfYC}(X')$  in phase 2 before contract  $x$ . In either case there exists  $z \in C_b^{BfYC}(X')$  such that  $z_i = y_i$ . Hence BfYC priorities are fair. QED

*Proof of Proposition 2*

Lemma 1 along with theorem 1 implies that the outcome of  $\varphi^{\text{ROTC}}$  is stable under ROTC priorities and it is weakly preferred by any cadet to any stable allocation. Lemma 1 along with theorem 2 implies that  $\varphi^{\text{ROTC}}$  is strategy-proof. Since cadets in the upper half of the OML priority ranking are denied eligibility for the last 35 percent of the slots at each branch under ROTC priorities, these priorities are not fair. Therefore,  $\varphi^{\text{ROTC}}$  is not fair either by proposition 1. Similarly,  $\varphi^{\text{ROTC}}$  does not respect improvements since a cadet can gain eligibility for the last 35 percent of the slots at each branch simply by lowering his OML priority ranking to the lower half. QED

*Proof of Proposition 3*

Lemma 2 along with theorem 1 implies that the outcome of  $\varphi^{\text{BfYC}}$  is stable under BfYC priorities and it is weakly preferred by any cadet to any stable allocation. Lemma 2 along with theorem 2 implies that  $\varphi^{\text{BfYC}}$  is strategy-proof.<sup>23</sup> Lemma 2 along with proposition 1 implies that  $\varphi^{\text{BfYC}}$  is fair.

All that remains is to show that  $\varphi^{\text{BfYC}}$  respects improvements. Fix a cadet  $i$  and let  $\pi_1$  be an unambiguous improvement for cadet  $i$  over  $\pi_2$ .

*Scenario 1:* First consider the outcome of  $\varphi^{\text{BfYC}}$  under priority order  $\pi_1$ . Recall that by remark 1, the order of cadets making offers has no impact on the outcome of the cumulative offer algorithm. Therefore, we can obtain the outcome of  $\varphi_{\pi_1}^{\text{BfYC}}$  as follows: First, entirely ignore cadet  $i$  and run the cumulative offers algorithm until it stops. Let  $X'$  be the resulting set of contracts. At this point, cadet  $i$  makes an offer for his first-choice contract  $x^1$ . His offer may cause a chain of rejections, which may eventually cause contract  $x^1$  to be rejected as well. If that happens, cadet  $i$  makes an offer for his second choice  $x^2$ , which may cause another chain of rejections, and so on. Let this process terminate after cadet  $i$  makes an offer for his  $k$ th-choice contract  $x^k$ . There may still be a chain of rejections after this offer, but it does not reach cadet  $i$  again. Hence cadet  $i$  receives his  $k$ th choice under  $\varphi_{\pi_1}^{\text{BfYC}}$ .

*Scenario 2:* Next consider the outcome of  $\varphi_{\pi_2}^{\text{BfYC}}$ , which can be obtained in a similar way: Initially ignore cadet  $i$  and run the cumulative offers algorithm until it stops. Since the only difference between the two scenarios is cadet  $i$ 's standing in the priority list,  $X'$  will again be the resulting set of contracts. Next cadet  $i$  makes an offer for his first-choice contract  $x^1$ . Since  $\pi_1$  is an unambiguous improvement for cadet  $i$  over  $\pi_2$ , precisely the same sequence of rejections will take place until he makes an offer for his  $k$ th-choice contract  $x^k$ . Therefore, cadet  $i$  cannot receive a better contract than his  $k$ th choice under  $\varphi_{\pi_2}^{\text{BfYC}}$  (although he can receive a worse contract if the rejection chain returns back to him). Hence  $\varphi^{\text{BfYC}}$  respects improvements. QED

<sup>23</sup> We could alternatively show that BfYC priorities are *substitutably completable*, a condition recently introduced by Hatfield and Kominers (2011), and use lemma 16 and theorems 17 and 18 in their paper to prove that  $\varphi^{\text{BfYC}}$  is strategy-proof.

*Proof of Proposition 4*

Suppose that priorities satisfy IRC, LAD, and the unilateral substitutes condition. Let  $\varphi$  denote the resulting COSM. By theorems 1 and 2,  $\varphi$  is stable and strategy-proof. To show the uniqueness, let  $\psi$  be a stable mechanism and suppose  $\psi \neq \varphi$ . Suppose that  $\psi$  is strategy-proof. We will show that this assumption results in a contradiction.

Since  $\psi \neq \varphi$ , there exists a preference profile  $P$  where  $\psi(P) \neq \varphi(P)$ . Let  $i$  be any cadet such that  $\psi(P; i) \neq \varphi(P; i)$ . Since cadet preferences are strict and mechanism  $\psi$  is stable, we have  $\varphi(P)P_i\psi(P)$  by theorem 1. This implies that  $\varphi(P; i) \neq \emptyset$ . Let  $\varphi(P; i) = (b, t)$ . Let  $P'_i \in \mathcal{P}$  be such that contract  $(i, b, t)$  is the only acceptable contract under  $P'_i$ . Since allocation  $\varphi$  is stable under  $P$ , it is also stable under  $(P'_i, P_{-i})$ . By theorem 6 of Hatfield and Kojima (2010), each cadet signs the same number of contracts at every stable allocation, and therefore,  $\psi(P'_i, P_{-i}; i) \neq \emptyset$ . Hence  $\psi(P'_i, P_{-i}; i) = (b, t)$ , and thus

$$\psi(P'_i, P_{-i}; i)P_i\psi(P; i),$$

where  $\psi(P'_i, P_{-i}; i) = \varphi(P; i) = (b, t)$ , contradicting the assumption that  $\psi$  is strategy-proof and completing the proof. QED

**Appendix B**

**Additional Excerpts on the Dead Zone**

The following forums posts from Service Academy forums (<http://www.serviceacademyforums.com/showthread.php?t=26435>) show that members of the ROTC community are well aware of some of the implications of the dead zone.

*Forum Post 1*

I just went through the accessions process this year. It went well for me, I accessed active duty and branched infantry, stationed at Ft. Drum. Both were my top choices. The only advice I can give on the subject is put in the time and effort early to avoid being disappointed when your a senior. Finally, DO NOT GET STUCK IN THE 30–40% on the active duty OML. Once in those percentiles it becomes mathematically impossible to get a competitive branch (ie. infantry and aviation). Hope this helps.

*Forum Post 2*

At first I thought you meant “bottom 30%–40%”, then I realized you meant 60th–70th percentile. OK, that is called the Dead Zone in Branch assignments. Not high enough to get 1st of 2nd Branch choice, but above the 50th percentile line, below which a lot of choice spots are reserved for “bottom halfers.” Strange sort of communism within our ranks. OH well, blame Congress.



Actually, per slide #5 of this deck: [http://www.career-satisfaction.army.ch\\_slides.html](http://www.career-satisfaction.army.ch_slides.html), the Dead Zone In the Active Duty Order of Merit List for 2010 cadets was:

Aviation: 22%–50%

Infantry: 26%–50%

Armor: 30%–50%

Medical Specialist: 38%–50%

Intelligence: 38%–50%

Note: 20% in the complete OML might actually be 28% in the “Active Duty” OML, so make sure you make this mental conversion to the complete OML during your first three years. Or, just really screw up everything except for GPA, and get yourself into the 55% (from the top = 45%) where you get your choice of Branch . . . just kidding. But in all seriousness, why create a system of merit evaluation that takes a top 40% OML cadet and rewards him/her for purposely sabotaging things to go DOWN in the OML to below the 50% AD OML line (just far enough to escape the DEAD ZONE) in order to get his/her choice of Branch?

This system must give rise to some really strange late night conversations among MSIII cadets:

*Cadet A:* “Hey, getting ready for LDAC? Brushing up on Night Navigation?. Are you still above 290 on your APFT?”

*Cadet B:* “Nah, I really want Infantry, I mean it’s all I’ve wanted since I was 5 years old, but I’m at the top 33% OML right now. I’ve got to screw up LDAC big time to drop down to 55% AD OML, so I can get Infantry out of the bottom half. I’m targeting a 260 APFT and I think I’ll just fail Night Nav. Oh yeah, and I’m dropping off the Club rugby team, cuz I don’t want those 2 PMS OML points awarded for sports participation”

*Cadet A:* “Yeah, but what if you miscalculate your gaming and end up at 48% AD OML? You’re not the only one trying to screw the pooch, you know. You have to adjust to how badly everybody else in the Infantry DEAD ZONE will also be screwing up at LDAC. You might need to mess something else up too”

*Cadet B:* “Oh, crap, didn’t think of that.”

### *Forum Post 3*

Its all a game of numbers. I know a couple of people who ended up getting “S”s at LDAC who got there first choice of branch because they were in the 51% percentile on the AD OML. Had they gotten an “E” they would have most likely been in the dead zone. Bottom line do your best and accept that you put forth your best effort. I watched many of my peers worry about branch day, but at the end of the day the Army needs good officers in all branches.

### *Forum Post 4 in Response to Forum Post 3*

Are you saying they should have tried to get E at LDAC, which would have put them at 46% AD OML, and then NOT gotten their choice of branch? I don’t disagree with you at all, but these kids in the DEAD ZONE are 20 year old cadets



being faced with a moral dilemma . . . do your best and kiss your branch choice goodbye, or screw up and get your Branch choice. I think this is a strange choice to put in front of a young cadet. Around 3,000 cadets commissioned and Branched AD that year, with about 200–250 in the DEAD ZONE prior to LDAC. So, while not a matter of National Security, I don't think the moral position these 200–250 cadets are put in is optimal.

*Forum Post 5 in Response to Forum Post 4*

Prior to going to LDAC it is almost impossible to know just how much you would need to “Tank” to secure a spot in the 50% area. This is a very risky thing to try and do. As the Army starts to cut back many of these cadets that try to work the numbers may find that they do not even make the AD cutoff line.

I talked a lot about this with my son before he left for LDAC, he didn't think anyone he knew in his battalion was even thinking about trying to gauge how they would end up on the OML by trying to slack off at LDAC, again just too risky.

My son did tell me about a cadet they knew at a cross town school. This cadet was determined to get Infantry, he knew he had to do poorly at LDAC since his grades were not very high, he just didn't know how poorly he had to do. This cadet did just that, he barely passed the APFT even though he had high scores at school, he did the bare minimum and just passed LDAC . . . barely. When the OML came out he was in that golden 50% spot and was confident he could now get Infantry, bragged about his ability to work the system. The thing this cadet forgot was that the PMS has to give his evaluation and can make comments regarding the cadets branch choice. Word got out that the PMS was not happy with his performance at LDAC, he knew the cadet could have done much better based on his performance at school. When the branches came out this cadet received Signal Corps, a great branch in my opinion, but not in his. It is believed that the comments from the PMS was the reason he did not receive Infantry, plus the low marks and APFT at LDAC didn't help either. Just goes to show, things can sometimes backfire.

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