Incentives

In many interactions agents have incentives to shirk, misrepresent preferences, etc. which leads to poor social outcomes. Some of these may be avoided by designing adequate contracts, tax mechanisms, etc.

How to Reward Work Effort?

Success of a project depends on effort of a worker.

- In case of SUCCESS the revenues are $200,000 whereas in case of FAILURE the revenues are $0.
- The worker can provide LOW or HIGH effort.
- The worker requires an expected salary of $50,000 to provide the LOW effort and $70,000 to provide the HIGH effort.
- The success probabilities are 80% for HIGH and 60% for LOW effort.

First assume that the effort can be observed.

- For LOW effort, the profits are

$$\Pi_L = 0.6 \times 200,000 - 50,000 = 70,000.$$

- For HIGH effort, the profits are

$$\Pi_H = 0.8 \times 200,000 - 70,000 = 90,000.$$

- Therefore the principal prefers the HIGH effort.
- In most applications the effort cannot be observed.
- In this case the worker has the incentives to pretend to be a HIGH effort worker although she is not. How can this problem be avoided?

- The key to answer is design of a contract where the salary consists of two parts: A base salary and a bonus in case of SUCCESS. Let's call these amounts x and y respectively.
- Therefore the contract is as follows:

$$\text{Salary} = \begin{cases} x & \text{in case of FAILURE} \\ x + y & \text{in case of SUCCESS} \end{cases}$$

- Let's find x and y. If the worker provides the LOW effort her expected salary is

$$0.6(x + y) + 0.4x = x + 0.6y$$

If the worker provides the HIGH effort her expected salary is

$$0.8(x + y) + 0.2x = x + 0.8y$$

Therefore the difference is

$$x + 0.8y - (x + 0.6y) = 0.2y$$

- On the other hand the difference the worker requires to provide the high effort level is 70,000-50,000 = 20,000. Therefore

$$0.2y = 20,000 \Rightarrow y = 100,000.$$

- What about the base salary x? Recall that the expected salary for HIGH effort is \$70,000. Therefore

$$0.8(x + 100,000) + 0.2x = 70,000$$
$$\Rightarrow x + 80,000 = 70,000 \Rightarrow x = -10,000$$

The base salary should be \$-10,000.

- Therefore the optimal contract is:

$$\text{Salary} = \begin{cases} -10,000 & \text{in case of FAILURE} \\ 90,000 & \text{in case of SUCCESS} \end{cases}$$

and the expected profits of the firm is

$$0.8 \times 200,000 - \text{Expected salary}$$
$$= 160,000 - 70,000 = 90,000$$

as in the case of observable effort level!

There are two potential problems here:

1. It may not be possible to punish the worker (with a fine of \$10,000) in case of failure. That is, it is possible that the base salary cannot be negative. In this case the base salary will be 0 and the expected salary of the worker will be $0.8 \times 100{,}000 =$ \$80,000. That will reduce the profits to \$80,000. In this case the inability to observe the effort level harms the firm.

2. We assumed that the worker is risk neutral. If the worker is risk averse then the firm may need to compensate the worker for the risk taken.

Pivotal Mechanisms

A local goverment will proceed with one of the two projects A or B. It wants to proceed with the one that maximizes the aggregate valuation. However voters have the incentives to exaggregate the valuations of their favorite projects. Hence the optimal project may not be selected.

Example: There are 5 voters:

Voter	A's worth	B's worth
1	20	15
2	0	-20
3	-15	-25
4	35	90
5	50	40
Total	90	100

Project B has larger aggregate worth but four voters prefer project A; therefore if the voters lie about their valuations project A may be selected. Can we avoid this incentives problem?

Consider the following **pivotal mechanism**:

1. Each voter reports a net valuation for both projects. The reported valuations can be true valuations or fake valuations.
2. The project with the highest total is chosen.
3. Each voter's report is deleted and replaced, one at a time, to see if that person's report affects the group decision.
4. In case a voter's report affects the group decision, the voter pays a tax equal to the difference between the aggregate valuations of the two projects without the preferences of the voter taken into account.

Example continued: Let's delete each voter's report and find the aggregate valuations:

Voter deleted	A's worth	B's worth
1	70	85
2	90	120
3	105	125
4	55	10
5	40	60

- Therefore in absence of voters 1,2,3, or 5 still the project B would be selected and hence their reports do not affect the group decision. As an implication they are not taxed.
- On the other hand in the absence of voter 4 the outcome A would be selected and therefore her report affects the group decision. The other voters' aggregate valuation of projects A and B are 55 and 10 respectively. Therefore voter 4 is taxed 55-10 = \$45.

Theorem: The pivotal mechanism eliminates the incentives to distort the true valuations.
In other words, truth-telling is a dominant strategy under the pivotal mechanism.

Let's illustrate this result with our example. Since voter 4 is the only one who pays tax, we start with her.

- Voter 4 is paying a tax of \$45. She can avoid this tax by reporting a higher valuation for A so that the project A is selected. But in this case her welfare reduces from 90 to 35, a loss of \$55. This is not profitable for her.

- Voter 1 is not paying any tax but he prefers the loosing project A. In his absence A's aggregate worth is 70 and B's aggregate worth is 85. Therefore in order to change the outcome to A, he should report a valuation for A that's at least 15 more than his reported valuation for B. In this case he'll affect the group decision and pay a tax of 85-70 = $15. But his benefits by changing the outcome is 20-15 = $5. Therefore misrepresentation is not profitable for him.

- Voter 2 is not paying any tax but he prefers A to B. In his absence A's worth is 90 and B's worth is 120. Therefore in order to change the outcome to A, he should report a valuation for A that's at least 30 more than his reported valuation for B. In this case he'll pay a tax of 120-90 = $30. But his benefits by changing the outcome is only 0-(-20) = $20.

- Voter 3 is not paying any tax but she prefers A to B. In her absence A's worth is 105 and B's worth is 125. In order to change the outcome, she should report a valuation for A that's at least 20 more than her reported valuation for B. In this case she'll pay a tax of 125-105 = \$20. But her benefits from changing the outcome is only -15-(-25) = \$10.

- Finally Voter 5 is not paying any tax but she prefers A to B. In her absence A's worth is 40 and B's worth is 60. Therefore in order to change the outcome to A, she should report a valuation for A that's at least 20 more than her reported valuation for B. In this case she'll pay a tax of 125-105 = \$20. But her benefits from changing the outcome is only 50-40 = \$10.

Second Price Sealed Bid Auction

Auctions are very popular in practice, especially when the market price of an asset is unclear. There are several types of auctions. The following is a very popular one:

The first-price sealed bid auction: There is one asset to be auctioned.

- Individuals submit sealed bids.
- The asset goes to the highest bidder who pays his own bid.

Although it is very popular, the first price auction has one important defect: Individuals have the incentives to underreport their valuations. How can we fix that problem?

Consider the following alternative auction:

The second-price sealed bid auction: There is one asset to be auctioned.

- Individuals submit sealed bids.
- The asset goes to the highest bidder who pays the *second highest* bid.

Theorem: Truth-telling is a dominant strategy under the second-price sealed bid auction.

Remark: In 1996 William Vickrey was awarded the Nobel prize for this invention.

Proof of the Theorem: We want to show that no individual can ever do better than reporting the true valuation.

Let's show this for an arbitrary individual. Let V be his true valuation and H be the highest bid of others. Let B denote the bid he is considering making. Suppose $B > V$. There are 3 possibilities:

- $B > V > H$: In this case bidding B or V doesn't matter. In either case he gets the asset and pays H.
- $H > B > V$: In this case too bidding B or V does not matter. In either case he gets nothing and pays nothing.
- $B > H > V$: If he bids V then he gets nothing and pays nothing. Therefore net gain from truthful bid is 0.
  If he bids B then he gets the asset and pays H. The net gain is $V - H < 0$. Thus he suffers a loss by bidding B.

Next suppose $B < V$. There are 3 possibilities again:

- $H < B < V$: In this case bidding B or V doesn't matter. In either case he gets the asset and pays H.
- $B < V < H$: In this case too bidding B or V does not matter. In either case he gets nothing and pays nothing.
- $B < H < V$: If he bids V then he gets the asset and pays H.
  The net gain is $V - H > 0$.
  If he bids B he gets nothing, pays nothing and his net gain is 0.
  In this case too he suffers by bidding B.

That takes care of all possibilities. Therefore there is no bid that can perform better than the truthful bid V.