Voting

- Suppose that the voters are voting on a single-dimensional issue. (Say 0 is extreme left and 100 is extreme right for example.) Each voter has a favorite point on the "spectrum" and the closer the current policy is towards their favorite point the better they are.

- Suppose that the outcome is determined by the mean of all voter's positions.

- In such an election voters have an incentive to misrepresent their preferences. For example if a voter's best point is to the right of the mean, he/she has the incentives to exaggregate that position so as to bring the mean closer to his/her best point. Since everyone has the same incentives the final outcome might be quite different than the mean of the true best points.

- *Question*: How can we solve this incentives problem?
- *Answer*: Choose the best point of the median voter (as opposed to the mean of the best points).
- **Median Voter Theorem:** Consider an election on a one-dimensional issue. Suppose each voter has a best point and the closer the outcome is to their best point the better they are. Suppose the (winner) outcome is the best point of the median voter. Then no voter has an incentive to distort his/her preferences. That is, truth-telling is a dominant strategy.
- The Median Voter Theorem is very nice but unfortunately it can be used only when the choices can be reduced to one dimension. In many cases that is not possible and therefore we need to consider other methods.

Majority Voting & Plurality Voting

- In situations where there are just two alternatives, **majority voting** (i.e. voting procedure where the winner is the alternative with the majority of the votes) is a very convenient voting method.
- It is simple and nobody has an incentive to misrepresent their preferences. That is, truth-telling is a dominant strategy under majority voting when there are two alternatives.
- When there are more than two alternatives a natural counterpart to majority voting is the **plurality voting** (i.e. voting procedure where the winner is the alternative with the plurality of the votes).
- Unfortunately truth-telling is no longer a dominant strategy in general under the plurality rule.

Example (Paradox of Voting): Suppose that the distribution of seats in a 100 member parliamentary assembly is (35,32,33) for Parties 1, 2, and 3 respectively. There are 3 alternatives A, B, C and the plurality rule is used to determine the outcome. The preferences of the parties are as follows:

Party 1	Party 2	Party 3
A	C	B
B	A	C
C	B	A

What will be the outcome in a secret ballot?

We can represent this secret ballot as a 3 person simultaneous game:

		Party 3								
		A			B			C		
		Party 2			Party 2			Party 2		
		A	B	C	A	B	C	A	B	C
	A	A	A	A	A	B	A	A	A	C
P1	B	A	B	B	B	B	B	B	B	C
	C	A	C	C	C	B	C	C	C	C

Let's solve this game by iteratively eliminating the dominated strategies. In this game

- Voting A is a dominant strategy for Party 1;
- Voting B is dominated by voting C for Party 2; and
- Voting A is dominated by voting B for Party 3.

Here note that while truth-telling is a dominant strategy for Party 1, it is not the case for Parties 2 and 3. After eliminating the dominated strategies the game reduces to:

		Party 3			
		B		C	
		A	C	A	C
Player 1	A	A	A	A	C

In this reduced game

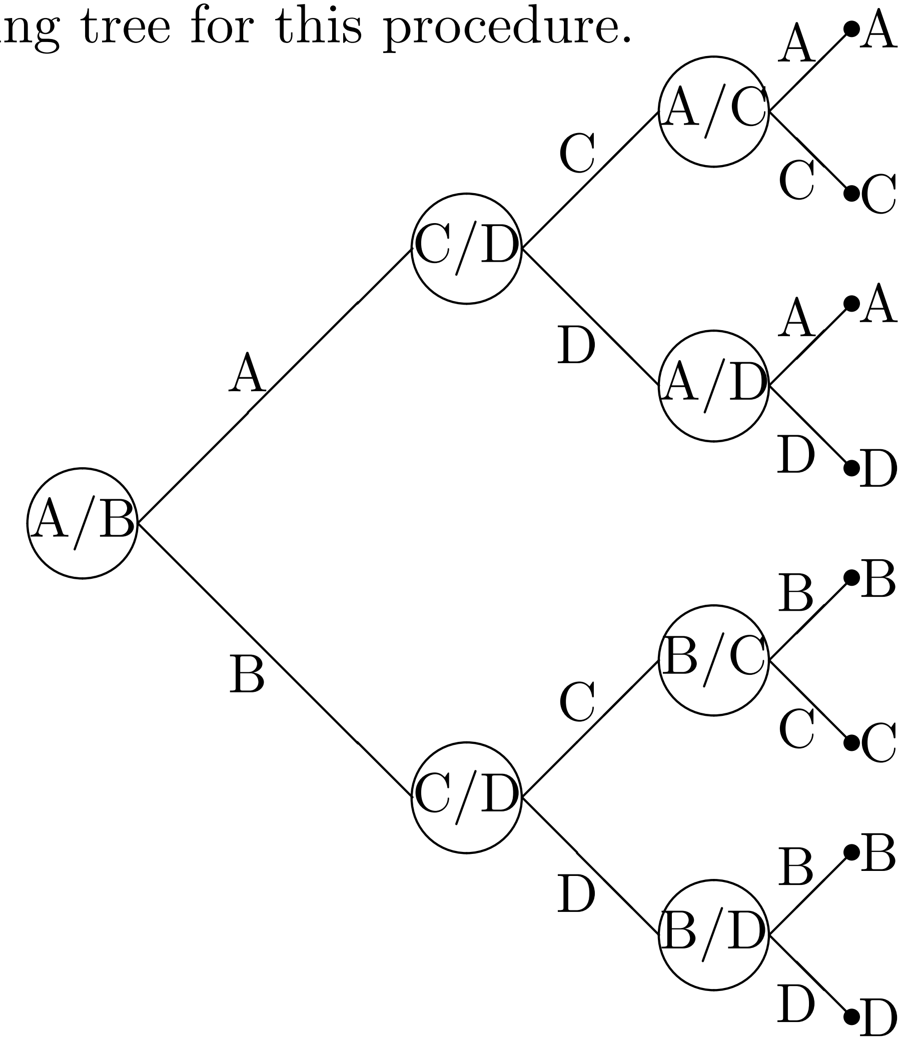
- C dominates A for Party 2, and
- C dominates B for Party 3.

Therefore Party 1 votes for A whereas Parties 2 and 3 vote for C and C is the winner! The paradox is that, C was the top choice of the smallest party and the last choice of the biggest party.

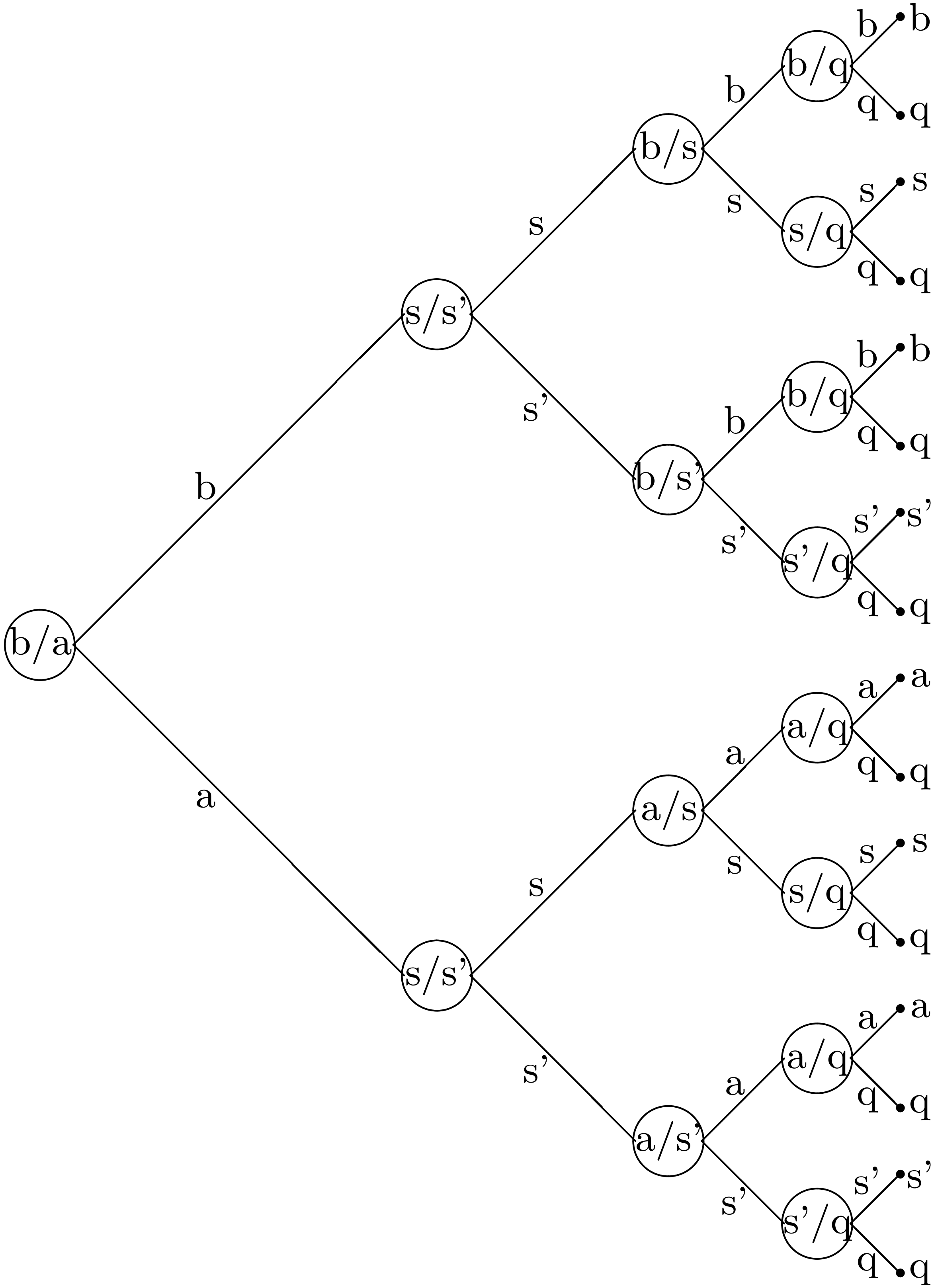
Voting Trees

- Suppose there are at least three alternatives. In such cases, while plurality rule is quite common, carrying out a number of pairwise competitions is even more popular. We can represent such procedures as extensive form games but that will be unnecessarilly complicated (especially when there are several voters).
- A **voting tree** is very useful in such situations. In a voting tree the terminal nodes represent the winning alternatives, the non-terminal nodes represent the votes, and the branches represent the winners of these votes.

For example suppose there are four alternatives A,B,C,D. In the first round A competes with B, in the second round C competes with D and in the last round the two winners compete. In each round the winner is determined by the majority rule. The following is the voting tree for this procedure.



Example: The following voting tree describes the usual procedure of the Congress when it confronts a bill (b), an ammended bill (a), a substitute bill (s), an ammended substitute (s'), and the status-quo (q). First the ammended bill a competes with the bill b, next the substitute bill s competes with the ammended substitute as, then the winners of the two, and finally the survivor against the status-quo q.


b/a
b
a
s/s'
s/s'
s
s'
s
s'
b/s
b/s'
a/s
a/s'
b
s
b
s'
a
s
a
s'
b/q
s/q
b/q
s'/q
a/q
s/q
a/q
s'/q
b
q
s
q
b
q
s'
q
a
q
s
q
a
q
s'
q

Sincere Voting

How do we solve these voting problems? That depends on whether the voters are strategic or not. Let's first assume that agents vote sincerely. That is at all stages agents vote for their most favorite choice. This is known as **sincere** (or **naive**) **voting**.

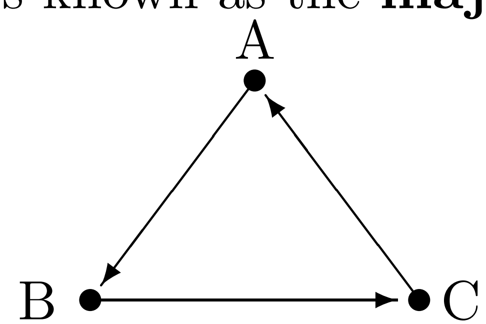
Example: There are three voters 1,2,3 and three alternatives A,B,C. Voter preferences (from best to worst) are as follows:

Voter 1	Voter 2	Voter 3
A	B	C
B	C	A
C	A	B

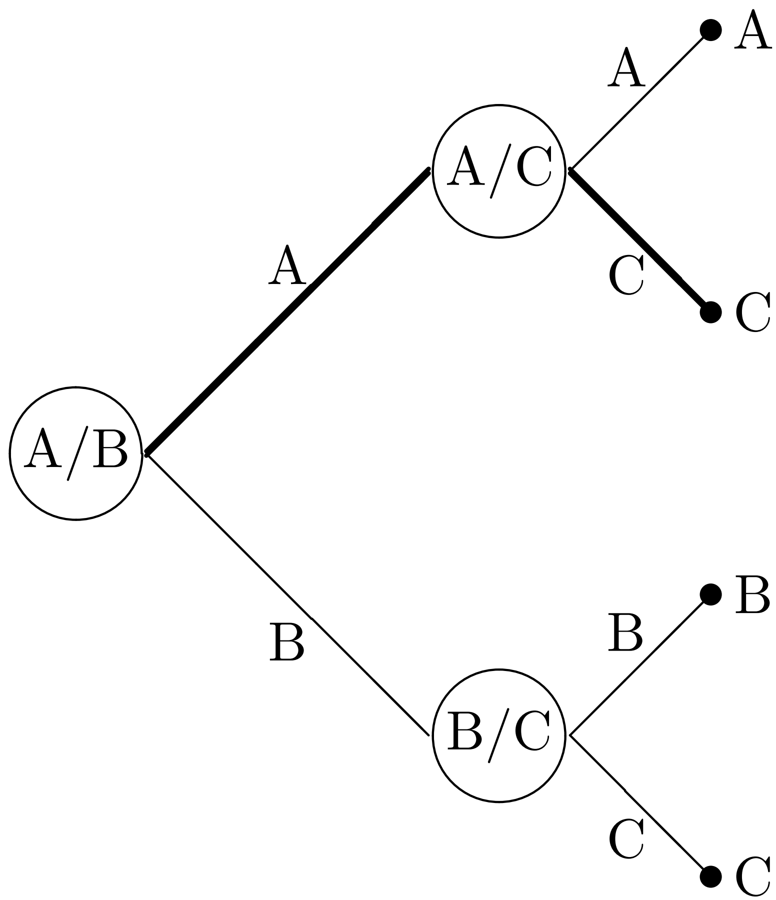
Let's find the sincere voting outcome for the following three procedures:

1. First A vs. B. Next the winner vs. C.
2. First A vs. C. Next the winner vs. B.
3. First B vs. C. Next the winner vs. C.

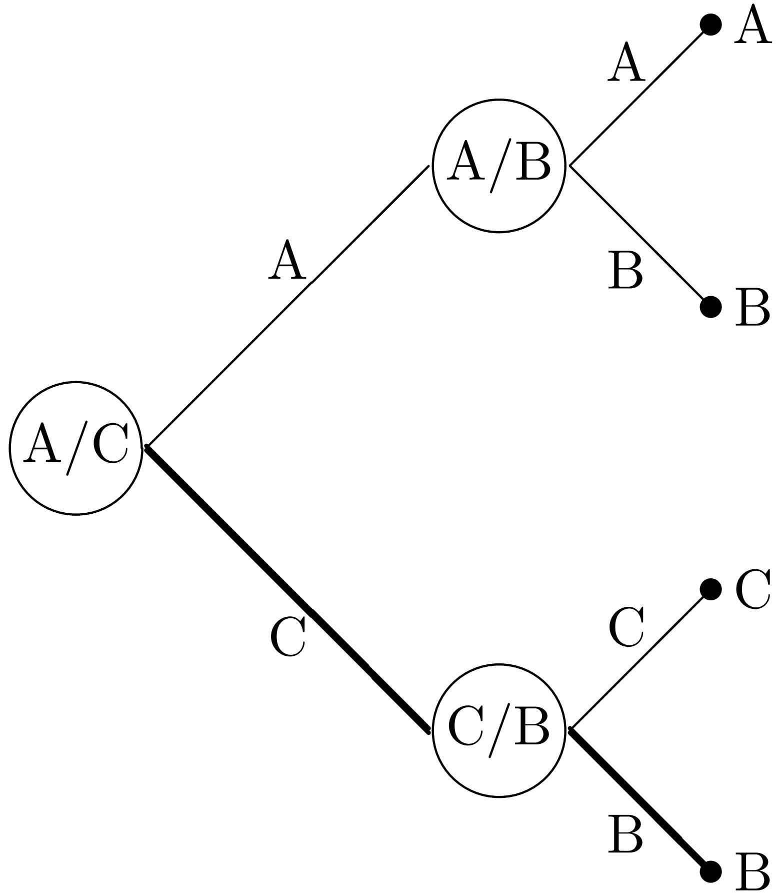
In voting problems obtaining the results of every pairwise competition is very useful. In this example A beats B (2 to 1), B beats C (2 to 1), and C beats A (2 to 1). We summarize this in the following diagram that's known as the **majority tournament**.



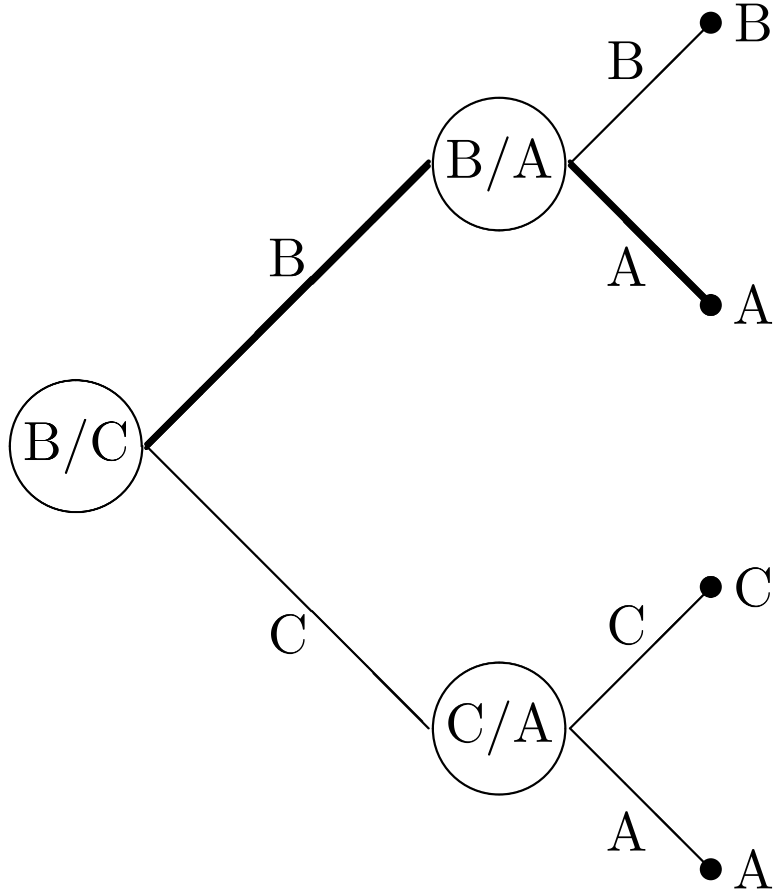
Voting Tree 1:



Voting Tree 2:



Voting Tree 3:



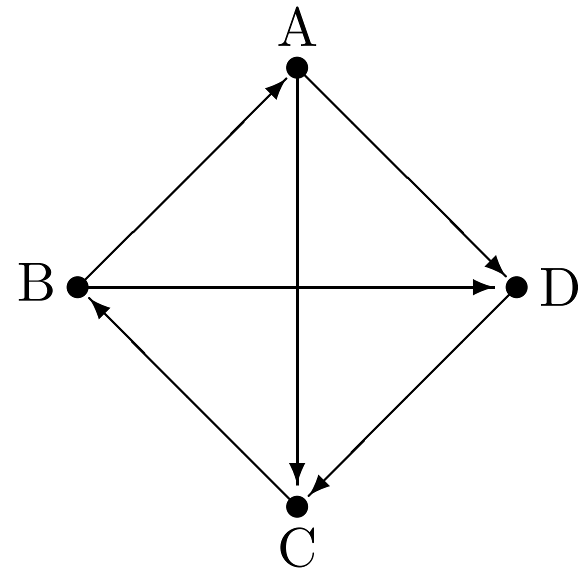
This example also shows that the order of the votes can be quite important. While the outcome is C under the first procedure, it is B under the second, and A under the third! **Agenda control** (or **agenda manipulation**) is the process of orginizing the order of the votes to assure favorable outcomes.

Example: There are three voters 1,2,3, and four alternatives A,B,C,D. Voter preferences are as follows:

Voter 1	Voter 2	Voter 3
B	A	C
A	D	B
D	C	A
C	B	D

Voting Procedure: First A vs. B, next the winner vs. C, and finally the winner vs. D.

Majority Tournament:



- Now it's easy to find the sincere voting outcome (even without drawing the voting tree). In the first round B beats A, in the second round C beats B, and in the last round D beats C and hence D is the final outcome.
- But there is something peculiar about this example. The alternative D is the winner and yet everyone prefers A to D! That is, D is not a **Pareto efficient** outcome: it is possible to make everyone better off.

Sophisticated Voting

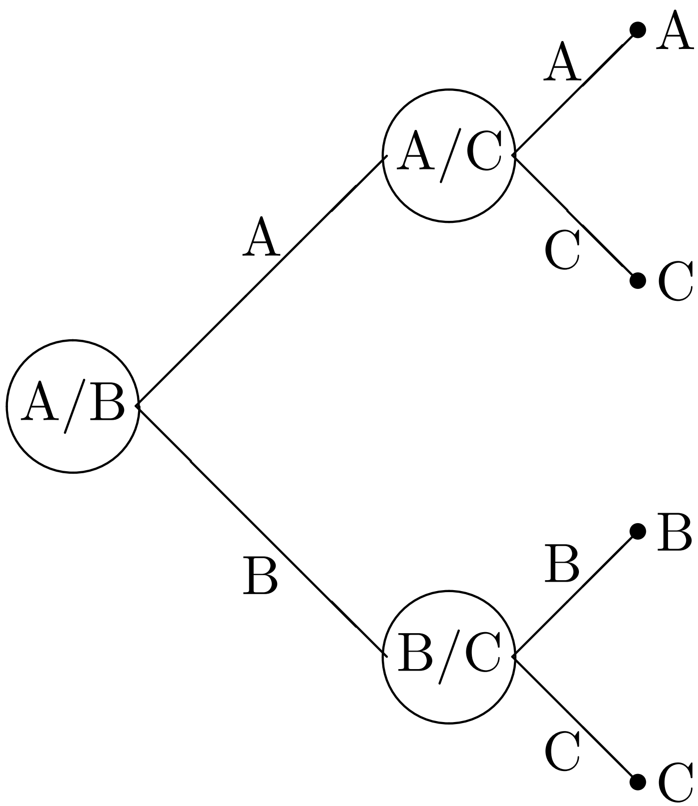
- In sincere voting it is assumed that all agents vote for the alternative they prefer at every vote.
- However in reality many voters would rather ignore the labels of the votes currently under consideration and focus instead on the consequences of each decision and vote for the alternative that yields the final outcome they most prefer. Such behavior is called **sophisticated** (or **strategic**) **voting**.
- Next we assume that everyone is sophisticated, everyone knows all preferences and everyone knows that everyone is sophisticated.

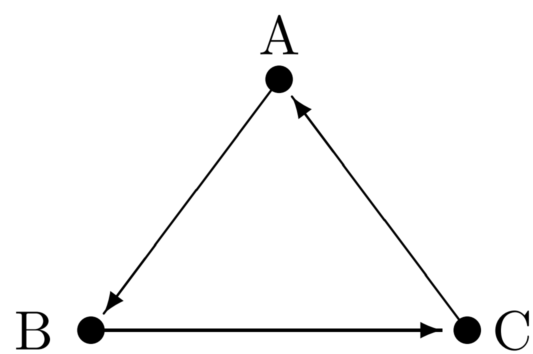
Example: There are three voters and three alternatives. Voter preferences are as follows:

Voter 1	Voter 2	Voter 3
A	B	C
B	C	A
C	A	B

Voting procedure: First A/B. Next winner/C.

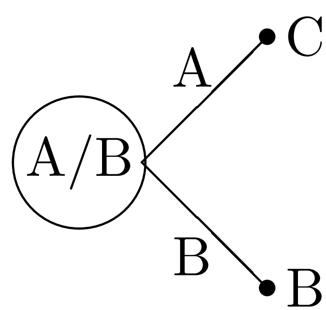
Voting tree & majority tournament:





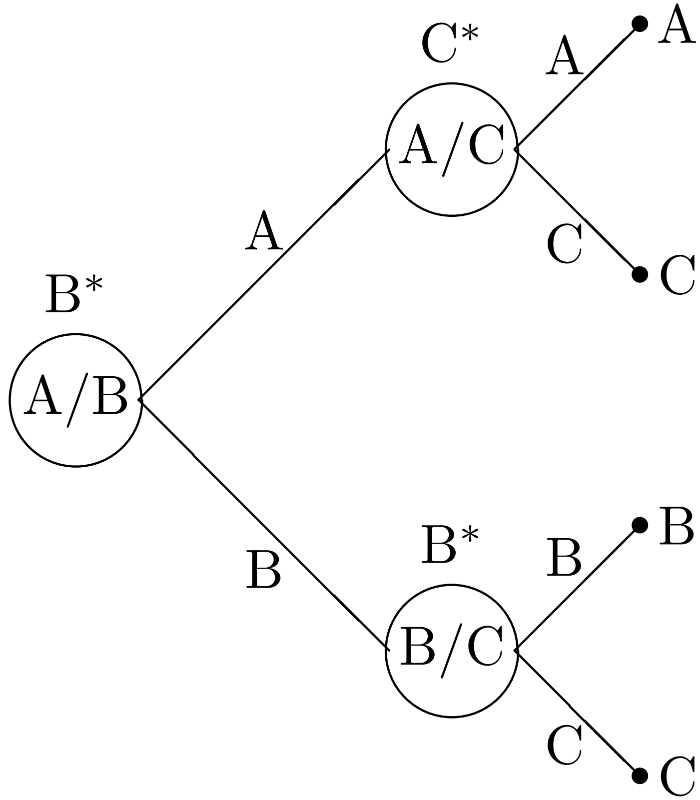
- Finding the sophisticated voting outcome is analogous to backwards induction. We start by solving the final round and move backwards.

- In the last round agents cannot gain anything by voting for the alternative they like less. Therefore every voter will vote sincerely in the final round. Hence in the vote A/C the outcome C will be the winner and in the vote B/C the outcome B will be the winner. Therefore the voting tree "reduces" to:



- In the first round (i.e. the vote A/B) voters understand that if they vote for A the eventual outcome will be C and if they vote for B the eventual outcome will be B.

- Therefore this is effectively a vote between C versus B! Since majority prefers B to C, the alternative B wins this vote and the eventual outcome is also B.
- We can represent this process by simply replacing the nodes representing votes with the eventual outcomes resulting from these votes:

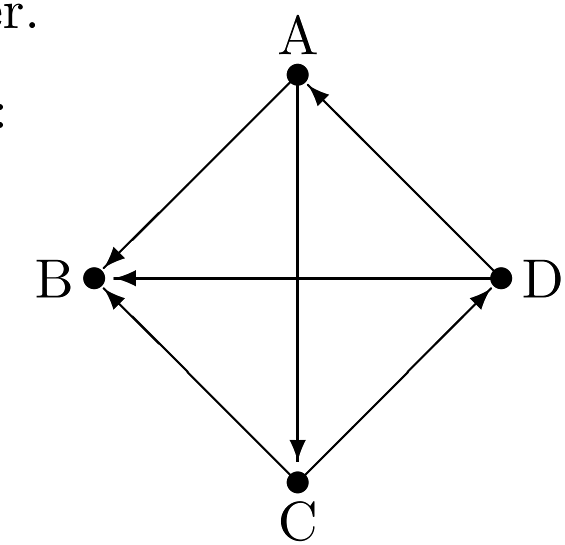


Example: There are four alternatives and five sophisticated voters with the following rankings:

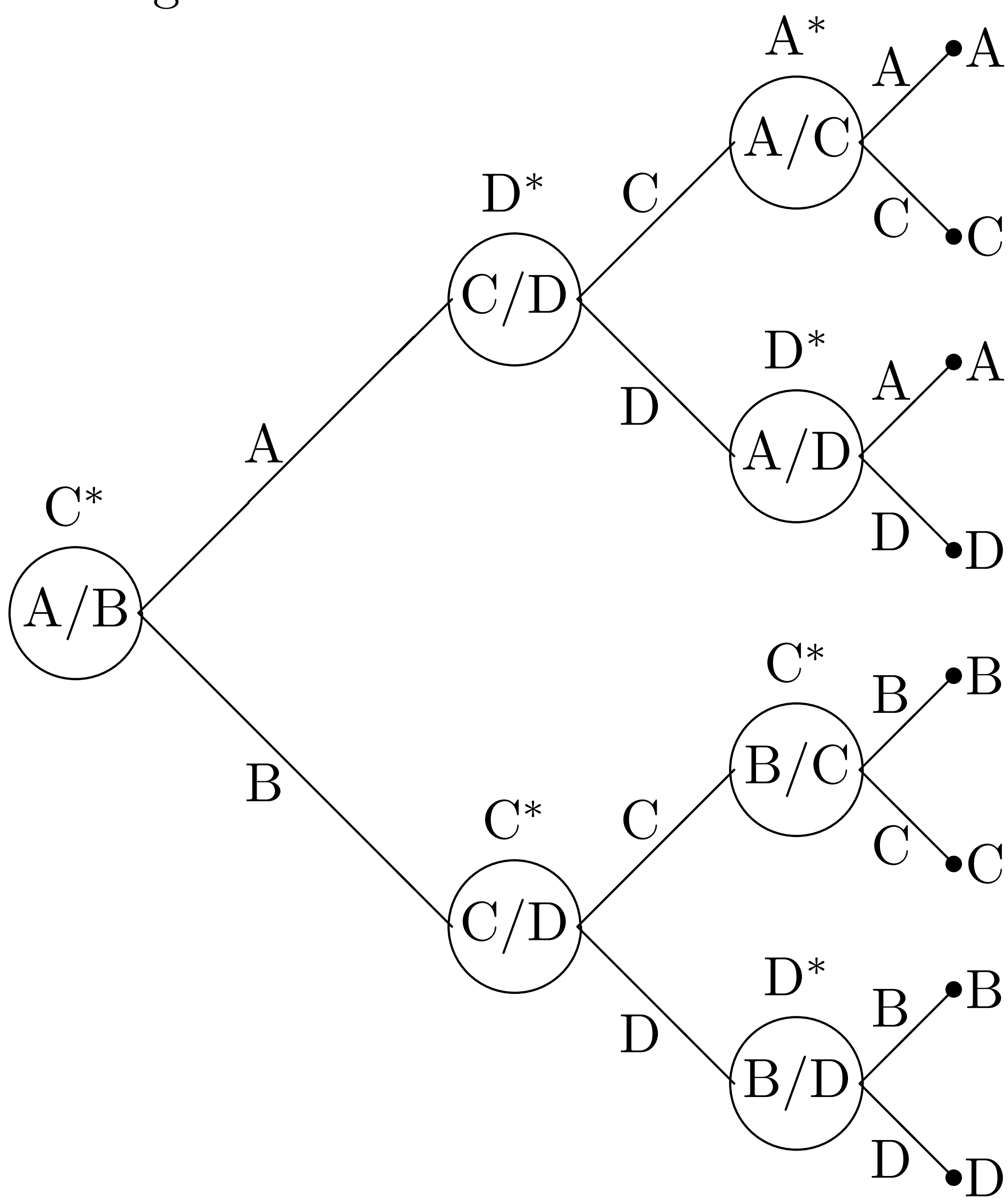
Voter 1	Voter 2	Voter 3	Voter 4	Voter 5
A	C	C	D	D
B	B	D	A	B
C	D	A	C	A
D	A	B	B	C

Voting procedure: First A vs. B; next C vs. D; and finally the two winners vs. each other.

Majority tournament:



Starting from the final round and moving backwards we find the sophisticated voting outcome to be C:

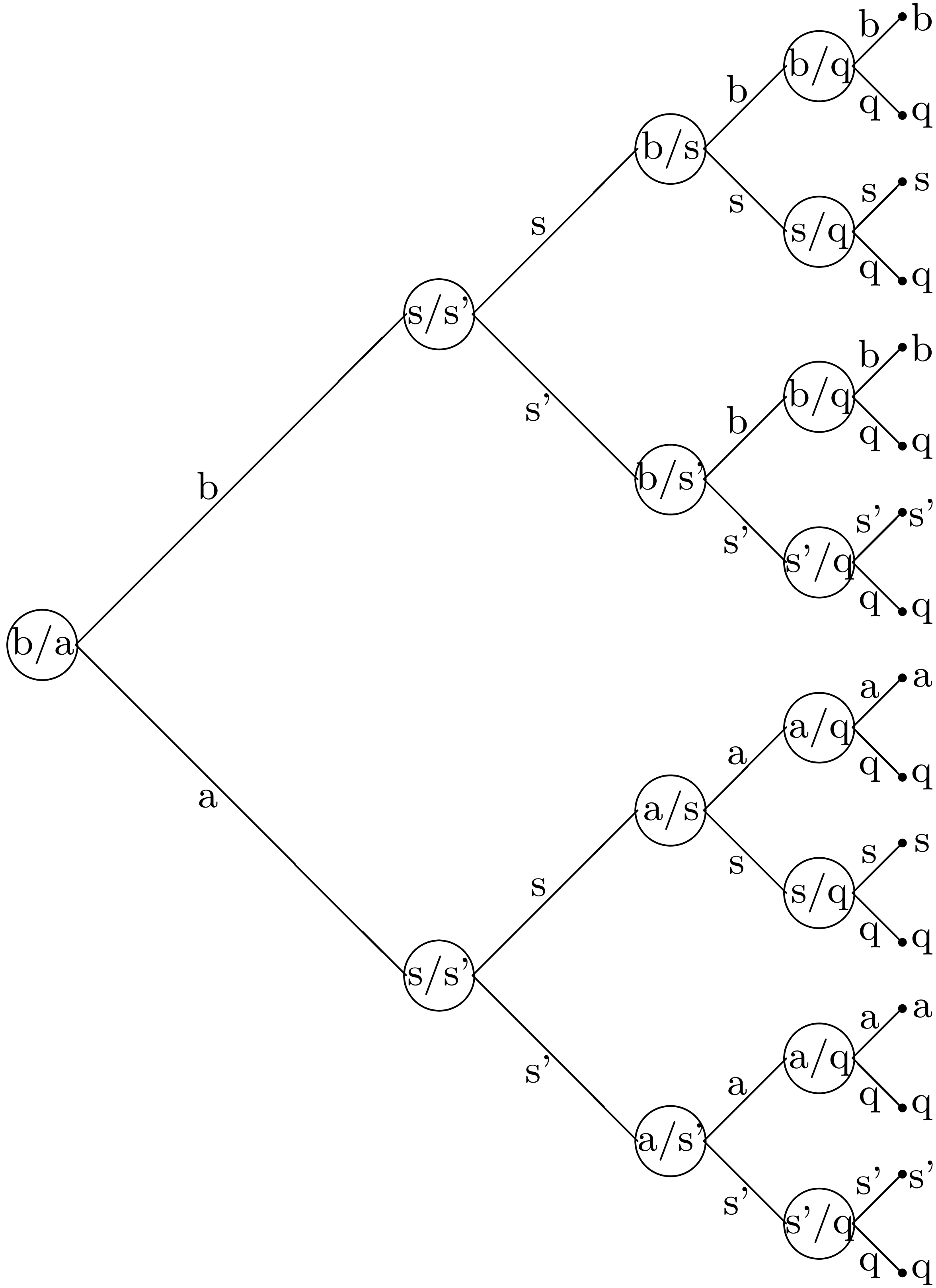


Example: Let's find the sophisticated voting outcome for the following example: There are five alternatives and three voters with the following rankings:

Voter 1	Voter 2	Voter 3
a	q	s'
b	b	s
s'	s	a
q	a	q
s	s'	b

Voting procedure:

- Round 1: b vs. a
- Round 2: s vs. s'
- Round 3: the two winners vs. each other
- Round 4: the survivor vs. q


b/a
b
a
s/s'
s/s'
s
s'
s
s'
b/s
b/s'
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Sophisticated & Sincere Voting Combined

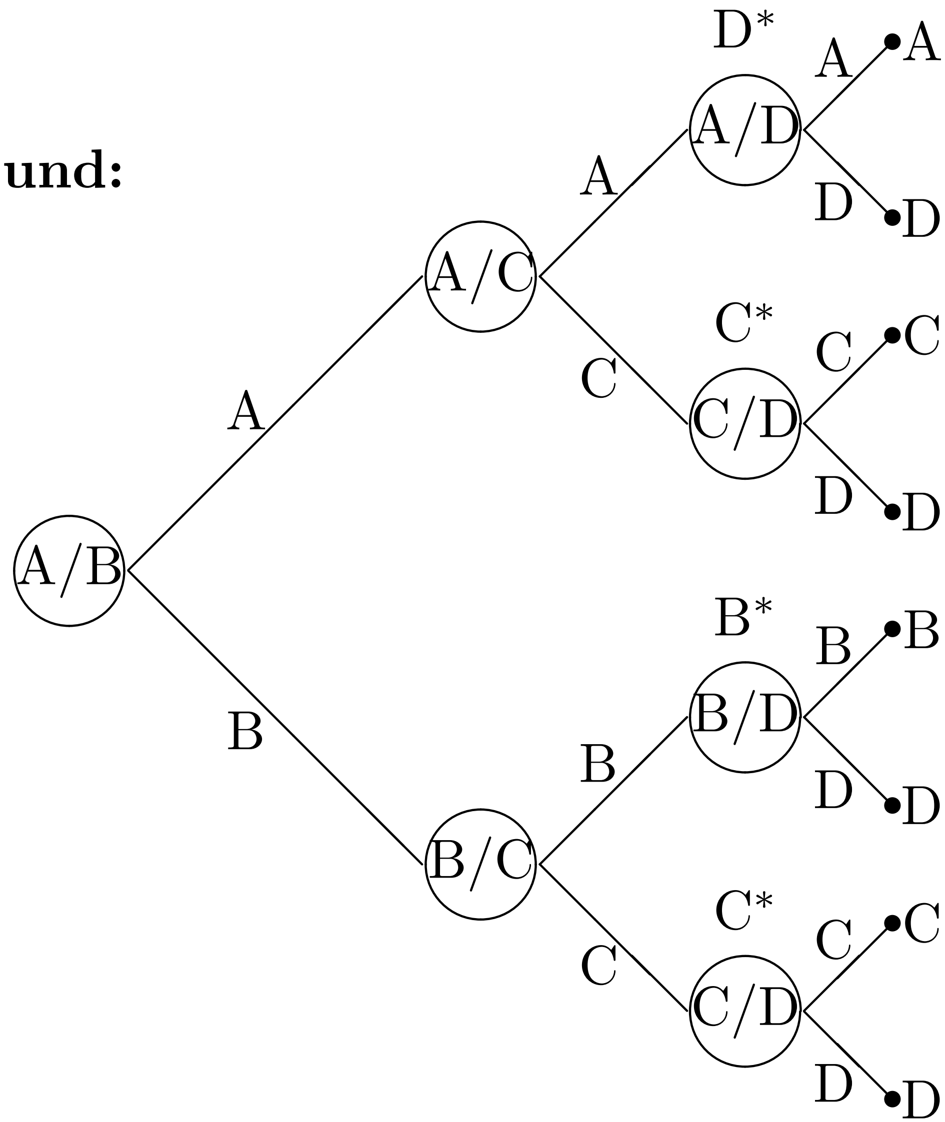
In this section we deal with situations where some voters are sincere and the others are sophisticated. We assume that sophisticated agents know who are sophisticated and who are not.

Example: There are four alternatives A, B, C, D, and five voters with the following rankings:

Voter 1	Voter 2	Voter 3	Voter 4	Voter 5
A	B	C	D	A
B	D	D	B	C
C	C	A	C	B
D	A	B	A	D

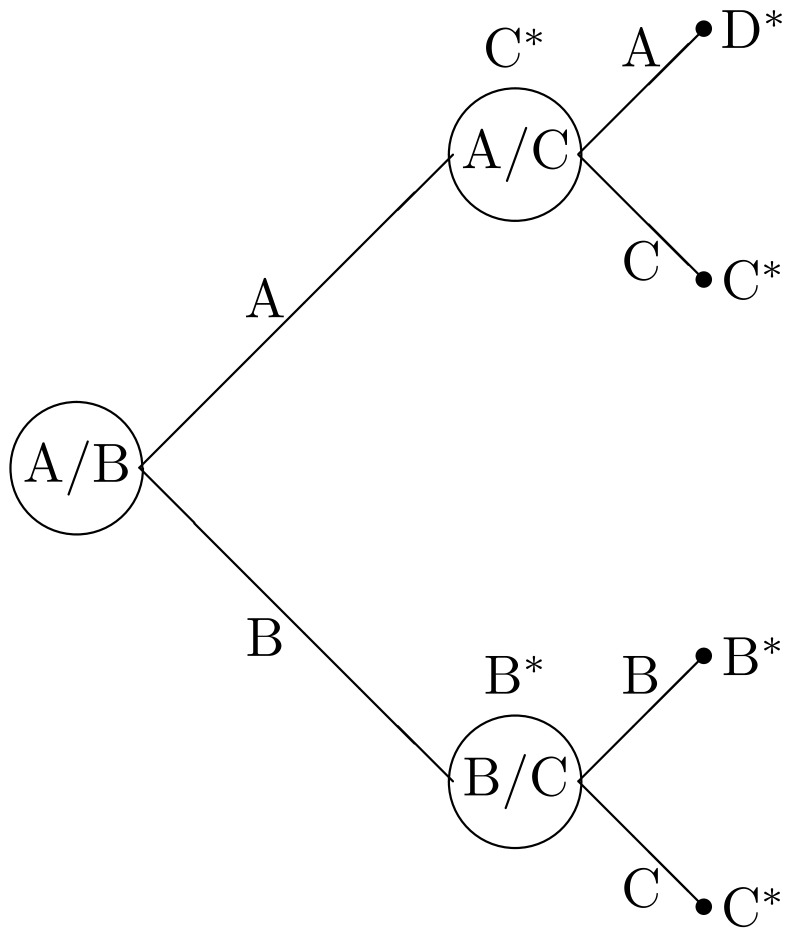
- *Voting procedure*: A versus B; winner versus C; winner versus D.
- Voters 1,2 are sophisticated and voters 3,4,5 are sincere.

Last Round:



In the last round everyone votes sincerely. Therefore winner of the vote A/D is D, winner of the vote C/D is C, and winner of the vote B/D is B.

Round 2:



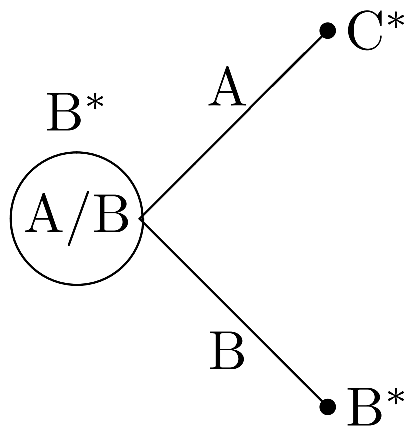
Vote A/C: Voters 1,2 view this vote as D/C whereas voters 3,4,5 view it as A/C.

- Voter 1 prefers C to D and hence votes for C.
- Voter 2 prefers D to C and therefore (although A is the last choice) votes for A.
- Voters 3,4 prefer C to A and vote for C.
- Voter 5 prefers A to C and hence votes for A.

C wins this vote. Moreover this vote eventually yields C as the eventual winner.

Vote B/C: Everybody views this vote as it is; B wins the vote and moreover this vote yields B as the eventual winner.

Round 1:



Vote A/B: Voters 1,2 view this vote as C/B whereas voters 3,4,5 view it as A/B.

- Voter 1 prefers B to C and therefore (although A is the top choice) votes for B.
- Voter 2 prefers B to C and hence votes for B.
- Voters 3,5 prefer A to B and hence vote for A.
- Voter 4 prefers B to A and hence votes for B.

B wins this vote and furthermore it is the eventual winner.

Example: There are four alternatives and three voters with the following rankings:

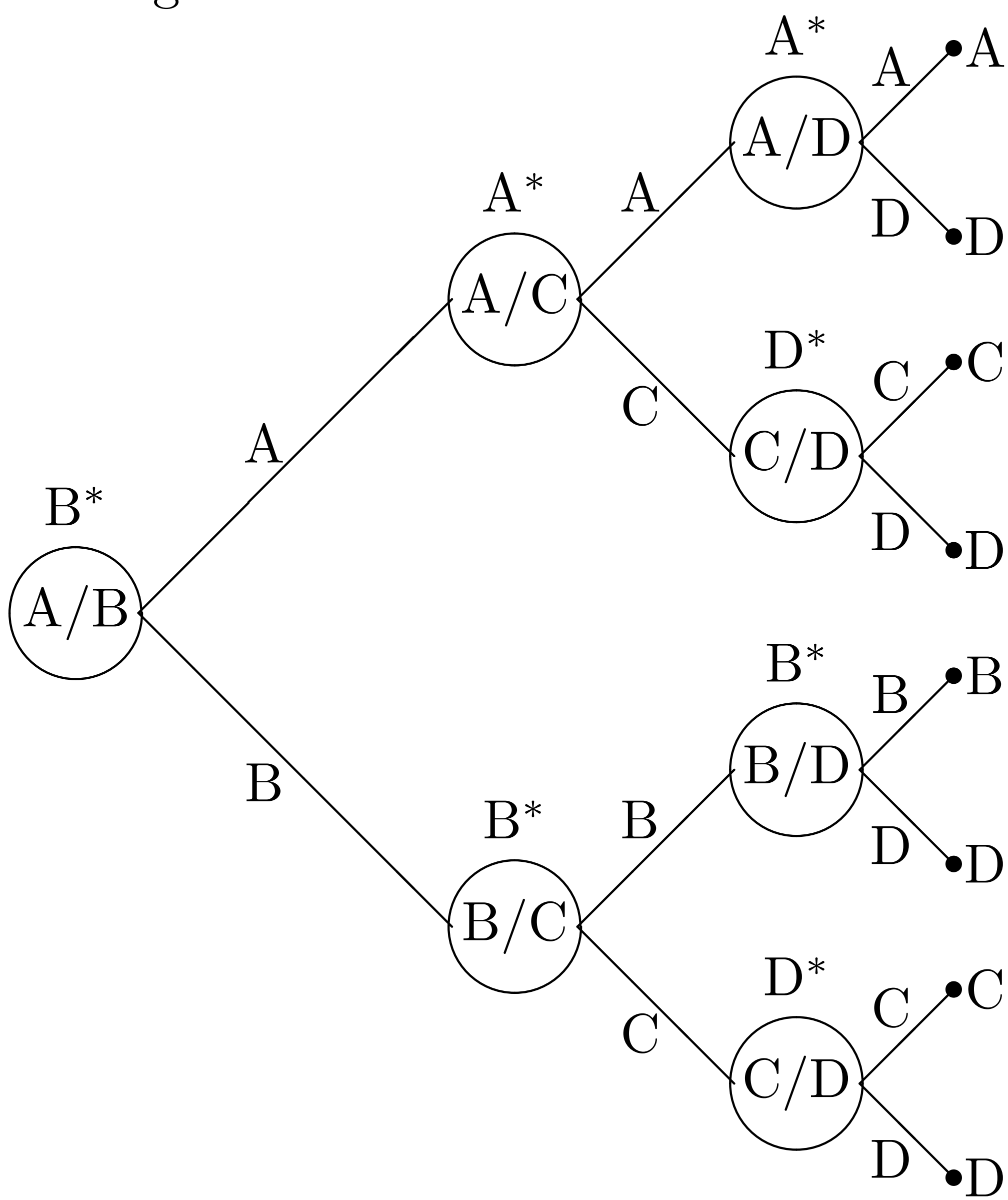
Voter 1	Voter 2	Voter 3
A	B	D
C	A	C
B	D	B
D	C	A

The voting procedure is as follows: A versus B; winner versus C; winner versus D.

We will compare the following 2 cases:

1. Everybody is sophisticated.
2. Voter 1 is sincere, voters 2 and 3 are sophisticated.

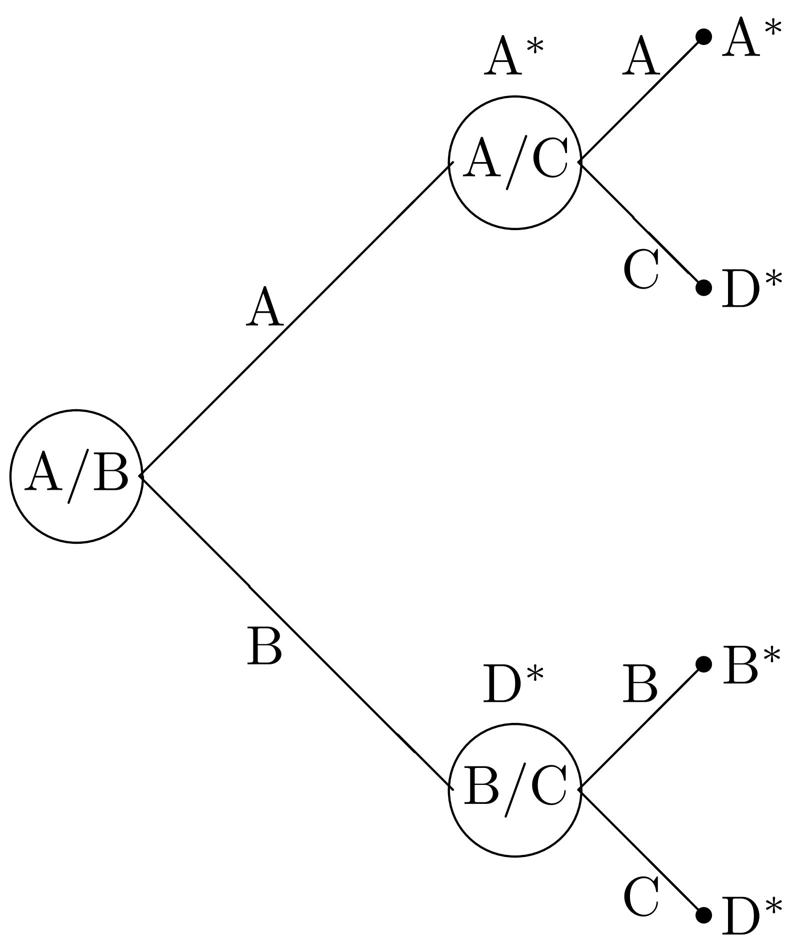
Starting from the last round and moving backwards we can find the sophisticated voting outcome to be B:



Next let's find the outcome when the first voter is sincere and the others are sophisticated.

Since everyone acts sincerely in the last round, agents vote as they do in the sophisticated voting. So we can start from Round 2.

Round 2:



Vote A/C: Voter 1 views this vote as A/C and voters 2,3 view it as A/D.

- Voter 1 prefers A to C and votes for A.
- Voter 2 prefers A to D and votes for A.
- Voter 3 prefers D to A and therefore votes for C.

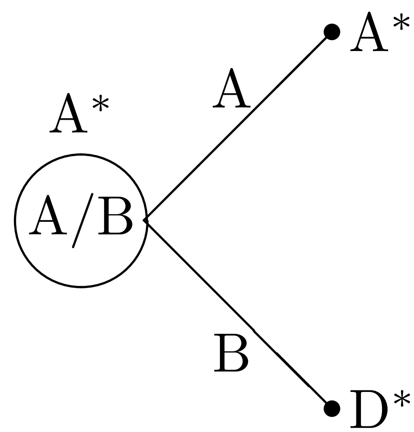
A wins the vote and it is the eventual outcome this vote leads.

Vote B/C: Voter 1 views this vote as B/C and voters 2,3 view it as B/D.

- Voter 1 prefers C to B and votes for C.
- Voter 2 prefers B to D and votes for B.
- Voter 3 prefers D to B and votes for C.

C wins this vote but D is the eventual outcome this vote leads.

Round 1:



Vote A/B: Voters 1 views this vote as A/B and voters 2,3 view it as A/D.

- Voter 1 prefers A to B and votes for A.
- Voter 2 prefers A to D and therefore (although B is the top choice) votes for A.
- Voter 3 prefers D to A and votes for B.

A wins this vote and also it is the eventual outcome.

This example shows that being sophisticated is not necessarilly beneficial!