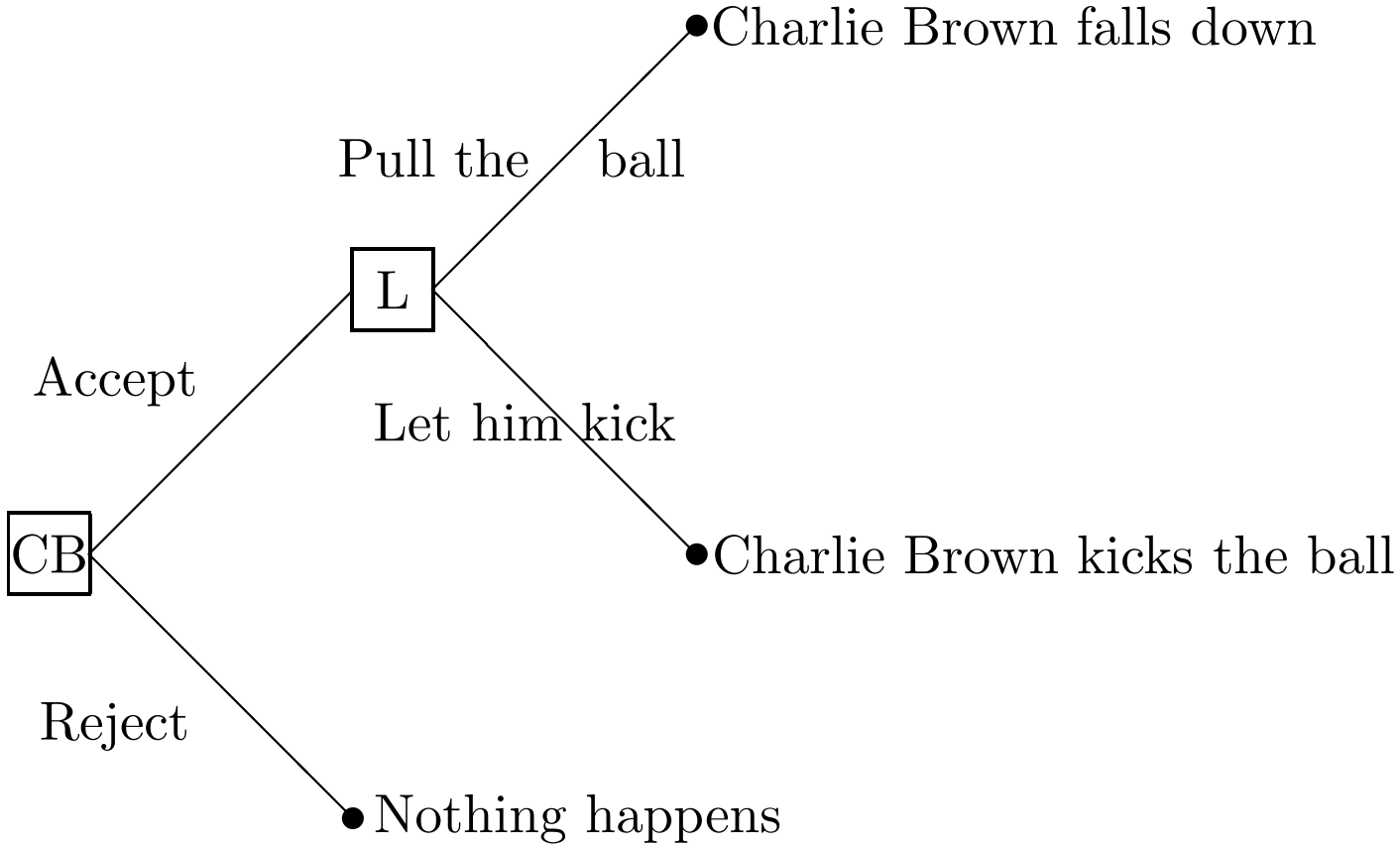
Sequential Games & Backwards Induction

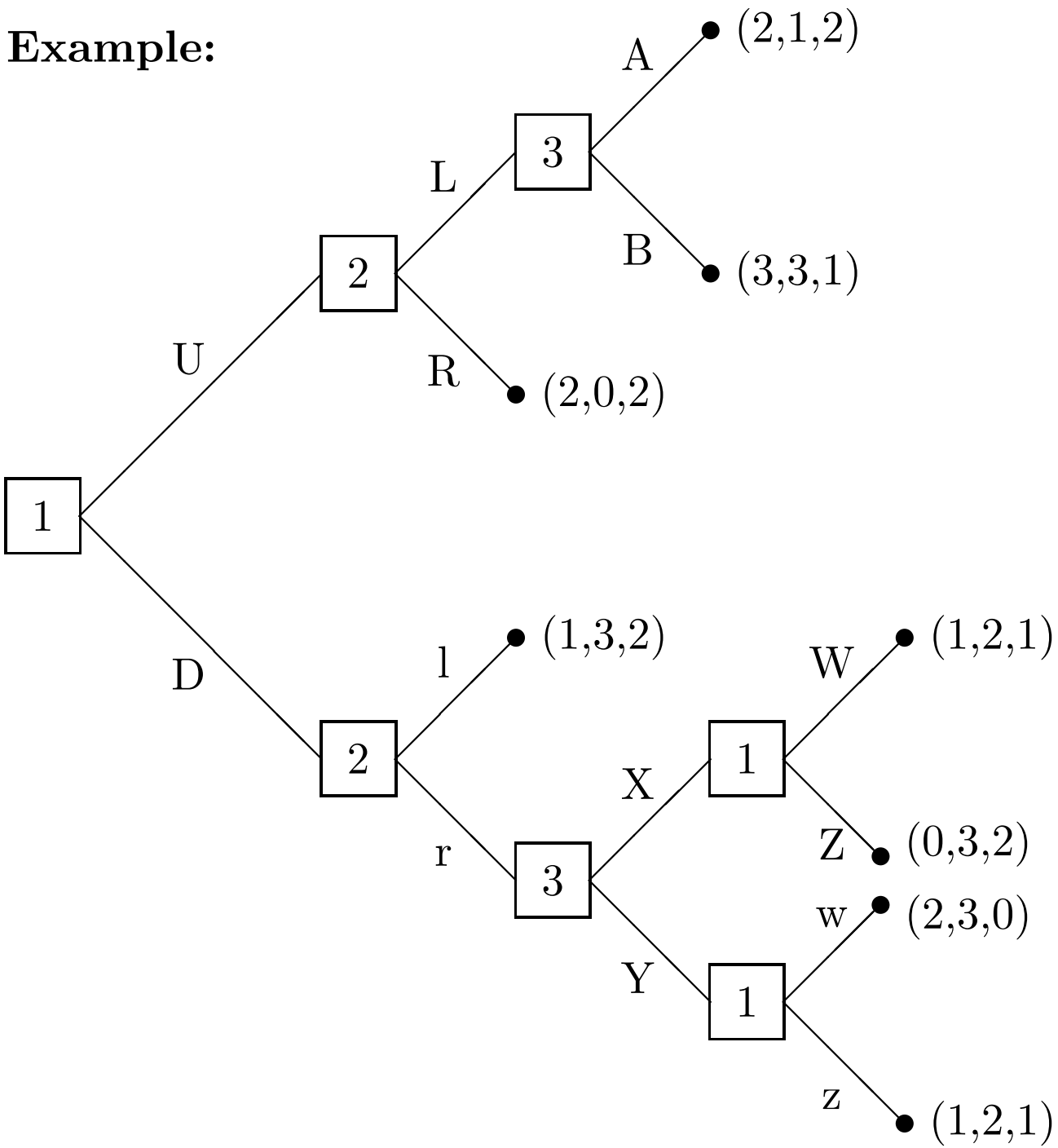
Solving a sequential game is easy. We begin with decisions that lead only to terminal nodes of the game, choose the action that maximizes the payoff of the player choosing at that node, and then work backwards through the earlier nodes of the game.

Each player can predict what other players will do at subsequent nodes, so he/she can predict the exact consequences of his/her possible moves before the move. For earlier nodes, we substitute the eventual outcome that will be reached for each move using the anticipated future moves of the other players. This is called **backwards induction**.

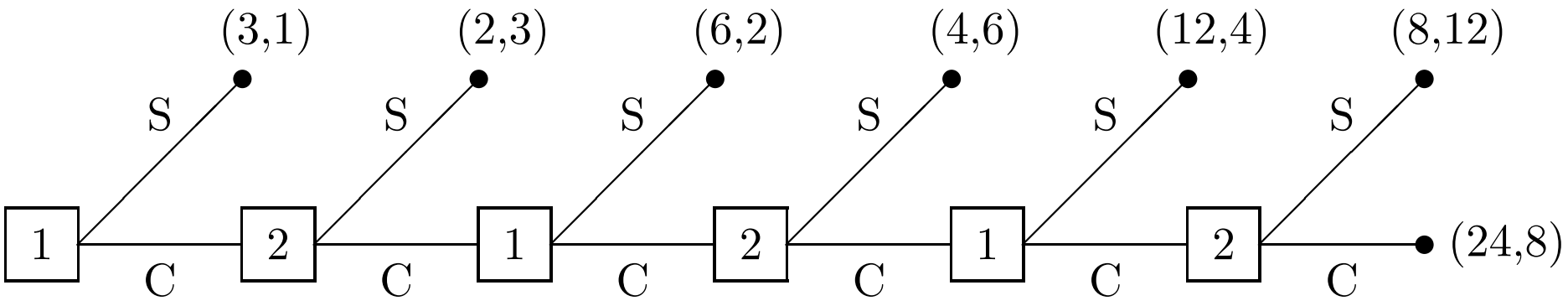
Example (Charlie Brown vs. Lucie): Suppose Lucie's top choice is Charlie Brown falling down, next choice is nothing happening, and last choice is Charlie Brown kicking the ball. Charlie Brown has the opposite ranking. Then we can solve the sequential game played by them using backwards induction:


Charlie Brown falls down
Pull the ball
L
Accept
Let him kick
CB
Charlie Brown kicks the ball
Reject
Nothing happens

Example:



Example (3-Level Centipede):



Example: Three members of a committee should choose among four candidates A,B,C,D. The procedure is such that

- member 1 vetoes a candidate,
- observing this member 2 vetoes another, and
- observing members 1 and 2, member 3 vetoes a third candidate.

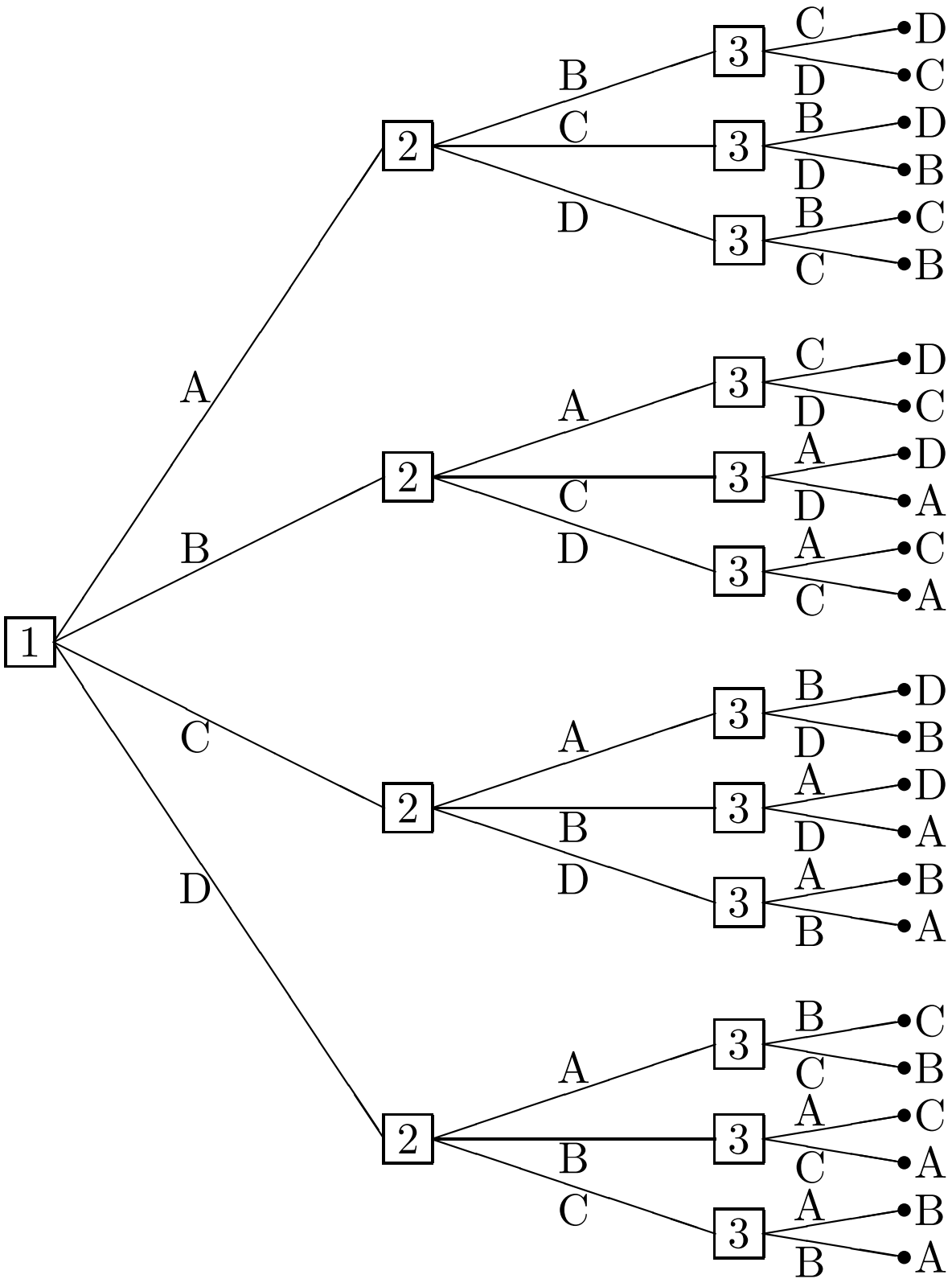
The surviving candidate is elected. The preferences of the members (from best to worst) are as follows:

Member 1: C B D A

Member 2: B C A D

Member 3: A B C D

Let's find the elected candidate using backwards induction.


1
A
B
C
D
2
B
C
D
A
C
D
A
B
D
A
B
C
3
C
D
D
C
B
D
D
B
B
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A
C
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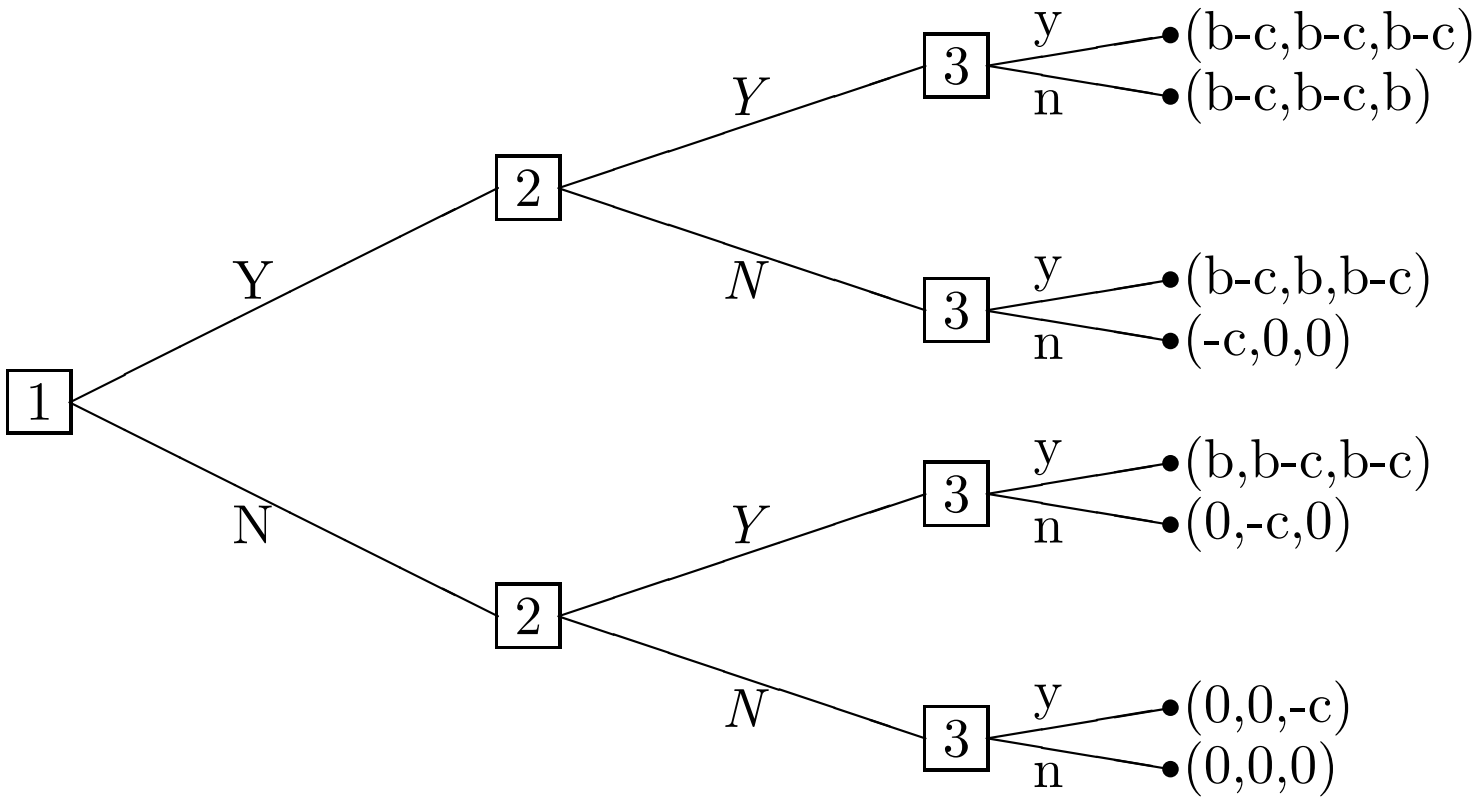
C
C
B
A
B
B
A

Example (Voting for a Pay Raise): Three legislators are voting on whether to give themselves a pay raise. All three want the pay raise; however each face a small cost in voter resentment $c > 0$. The benefit for the raise is $b > c$. They vote in the order 1-2-3. What is the outcome obtained by backwards induction?



Example (Tricky!):