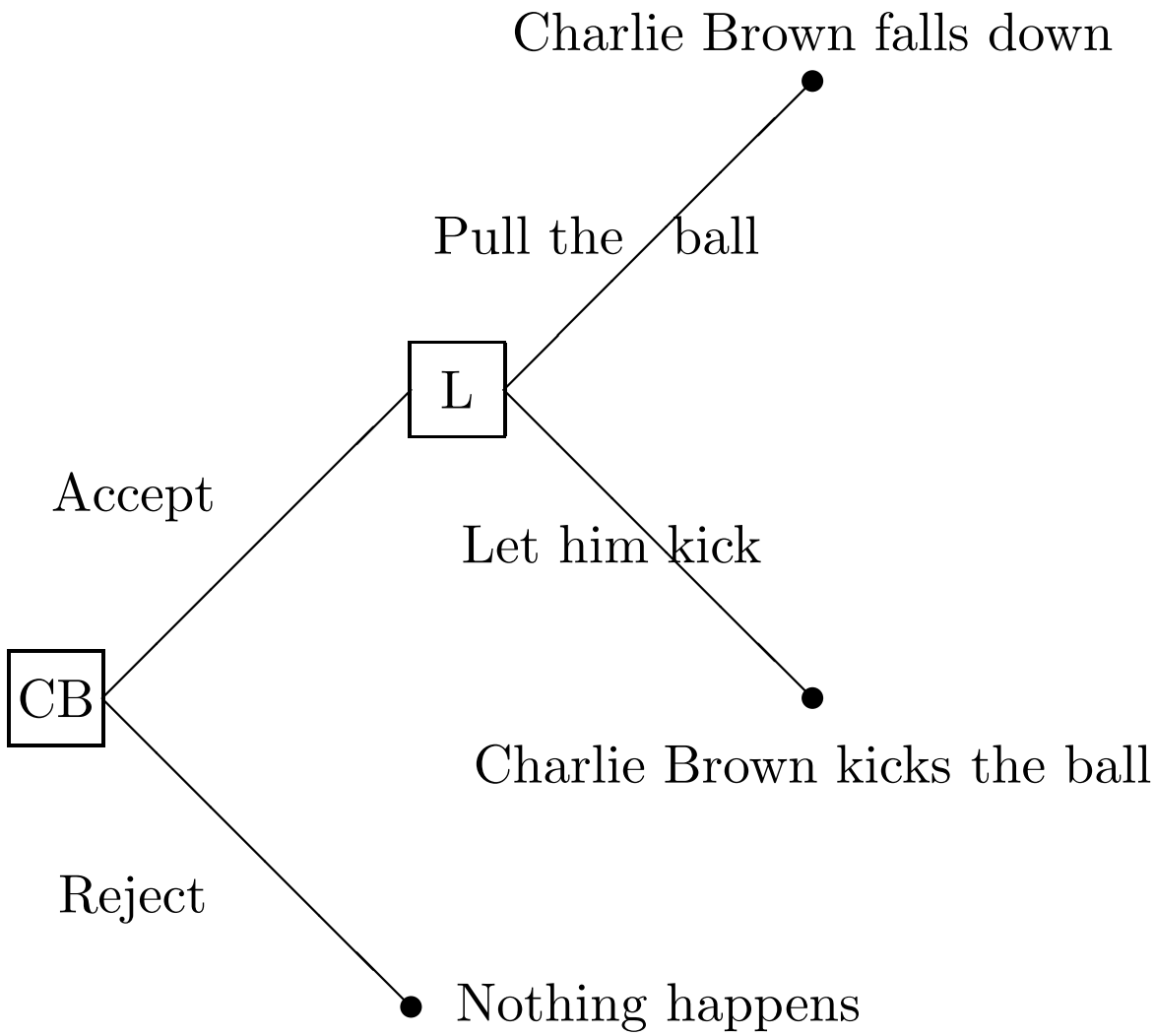
Strategic & Extensive-Form Games

Here Charlie Brown, knowing that Lucie would very much like to see him falling down, should have never agreed!

There are two basic types of interactions:

- **Sequential**: Players make alternating moves
- **Simultaneous:** Players act at the same time

The interaction in the cartoon is purely sequential and we can represent it as follows:


Charlie Brown falls down
Pull the ball
L
Accept
Let him kick
CB
Charlie Brown kicks the ball
Reject
Nothing happens

In most cases interactions are partly sequential and partly simultaneous. We can model such interactions in two ways:

1. **Strategic Games**
2. **Extensive Form Games**

Strategic Games

Easier or the two models.

This model is *mostly* (but not exclusively) for *simultaneous* interactions. (Therefore all simultaneous games that we will study are strategic games)

There are three components in a strategic game:

1. A list of participants, or **players**
2. For each player, a list of **strategies**
3. For each array of strategies (one for each player), a list of **payoffs** that the players receive

Example (Rock, Paper, Scissors):

		Player 2		
		Rock	Paper	Scissors
Player 1	ROCK	0,0	-1,1	1,-1
	PAPER	1,-1	0,0	-1,1
	SCISSORS	-1,1	1,-1	0,0

- Since there are two players, and each has three strategies, the set of **strategy profiles** forms a 3×3 table.
- We list Player 1's strategies as rows in the table, and Player 2's strategies as columns.
- For each of $3 \times 3 = 9$ cells in the table, the first payoff is to Player 1, the second one is to Player 2.

Example (A 3 Person Strategic Game): In the following 3 person simultaneous game player 1 chooses the row, player 2 chooses the column, and player 3 chooses the box.

Player 3

A

Player 1		Player 2 L	Player 2 R
	U	1,0,2	1,1,1
	M	2,0,2	1,1,1
	D	1,1,1	0,2,2

B

		Player 2 L	Player 2 R
	U	1,0,0	1,2,1
	M	0,2,2	1,1,1
	D	1,1,1	2,0,2

Extensive-Form Games

In an extensive form game, attention is given to

1. the *timing* of the actions that players may take, and
2. the *information* they have when they must take those actions.

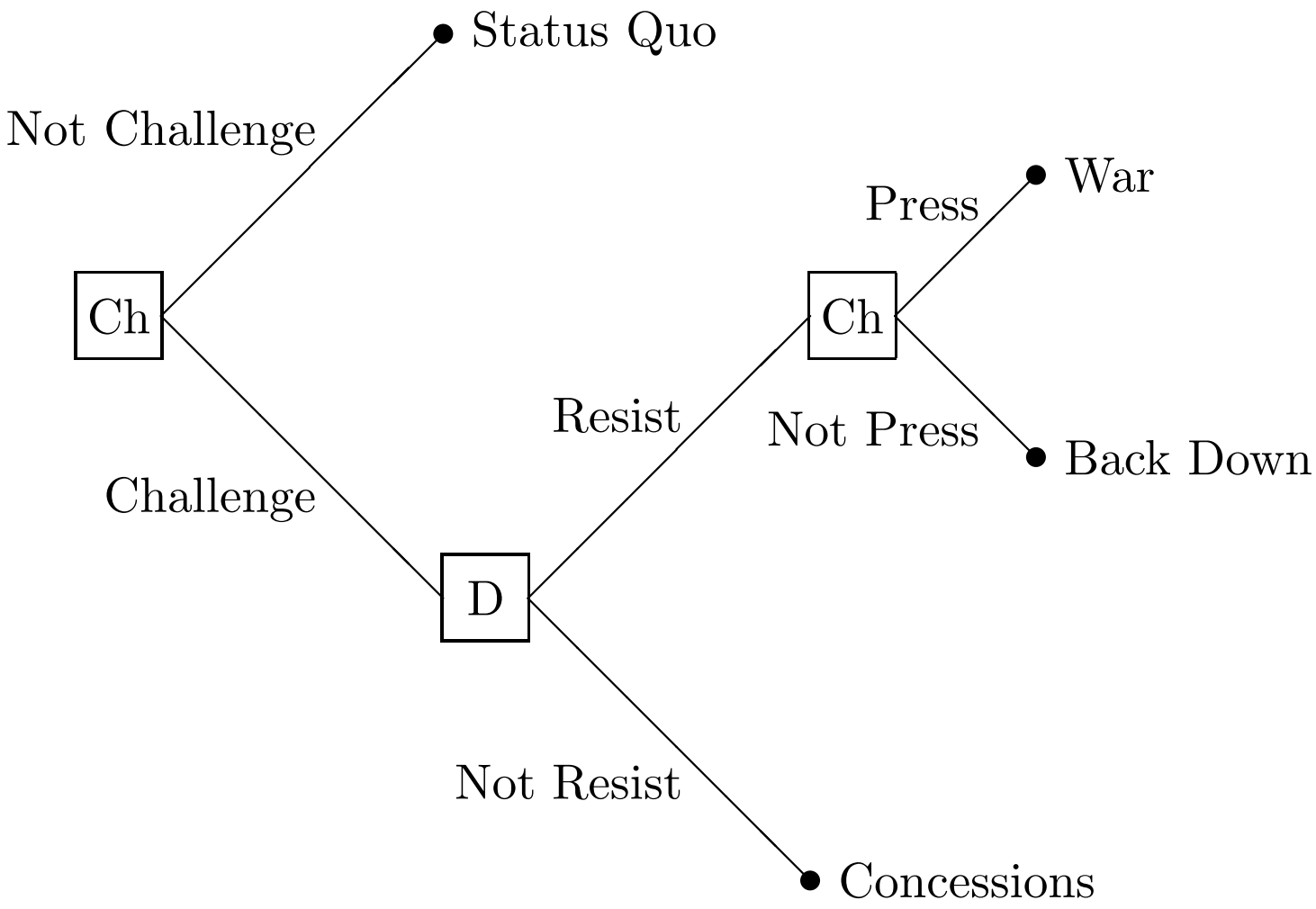
Before we give the formal definitions, let's give several detailed examples.

Example (Deterrence in the Cuban Missile Crisis):

- Why did the Soviet Union attempt to place offensive missiles in Cuba?
- Why did US respond with a blockade of Cuba?
- Why did the Soviet Union decide to withdraw the missiles?

Let's treat this crisis as a contest between two players, a Challenger (Player Ch) and a Defender (Player D).

How can we model such a contest?


Status Quo
Not Challenge
War
Press
Ch
Ch
Resist
Not Press
Back Down
Challenge
D
Not Resist
Concessions

Preferences

Challenger: $C \quad \underbrace{SQ \quad BD}_{W}$

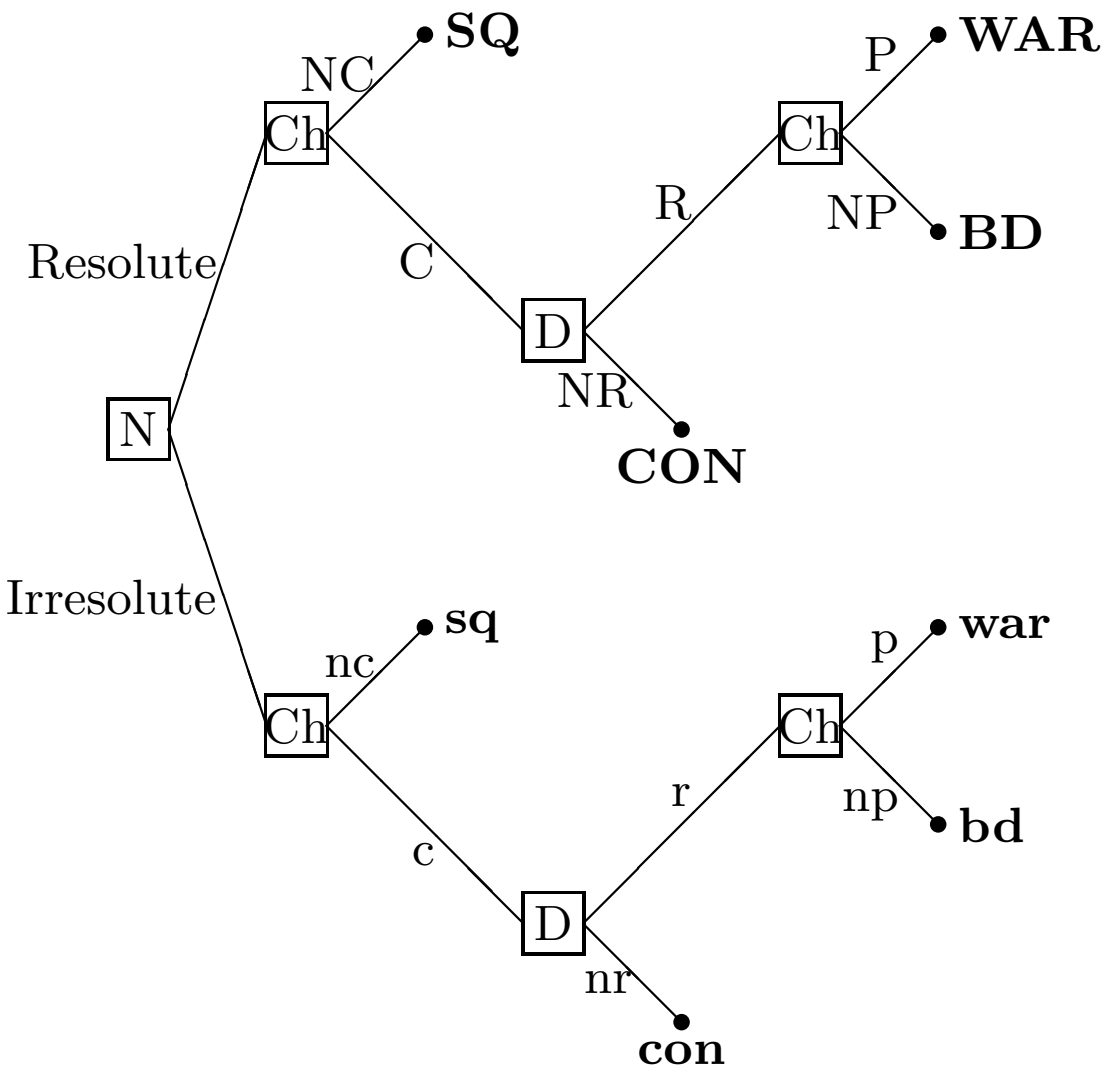
Defender: $BD \quad \underbrace{SQ \quad C}_{W}$

We can (and usually do) assign **payoffs** for each player for these outcomes.

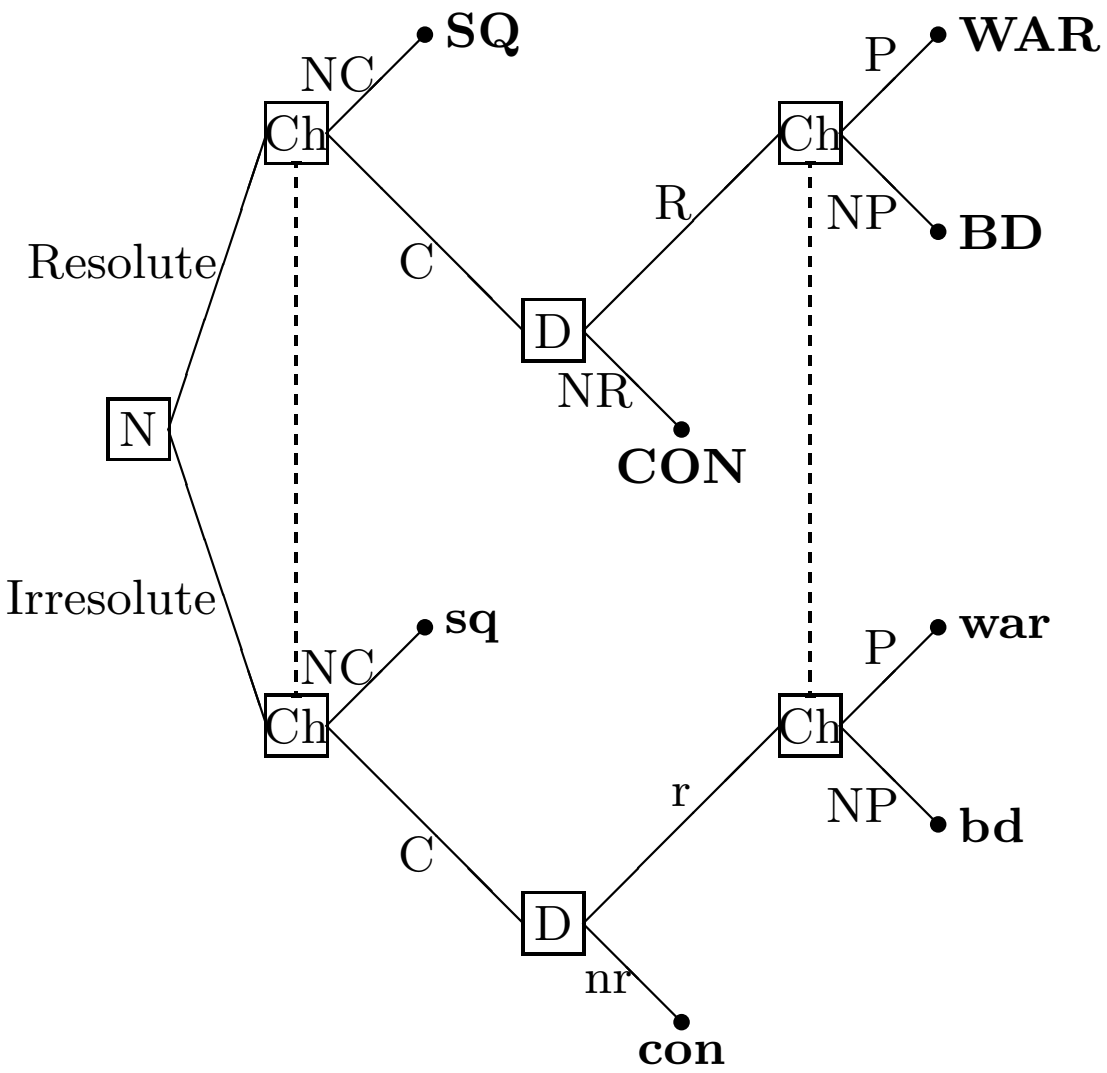
We assume that all players know others' payoffs. Any information is **common knowledge**. If there is **uncertainty**, we can model this by adding **Nature** (or **Chance**) as another player (who does not have payoffs) and it chooses different types. For example suppose there are two types of Defenders:

- Resolute type who prefers War to Concessions
- Irresolute who prefers Concessions to War

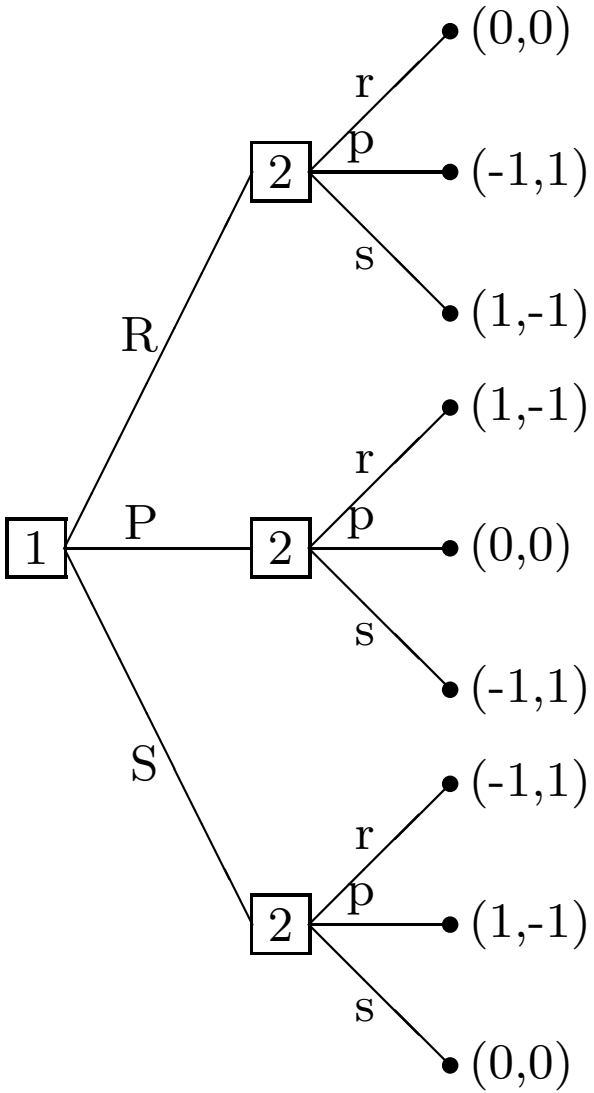
If the Challenger can observe the Defender type:



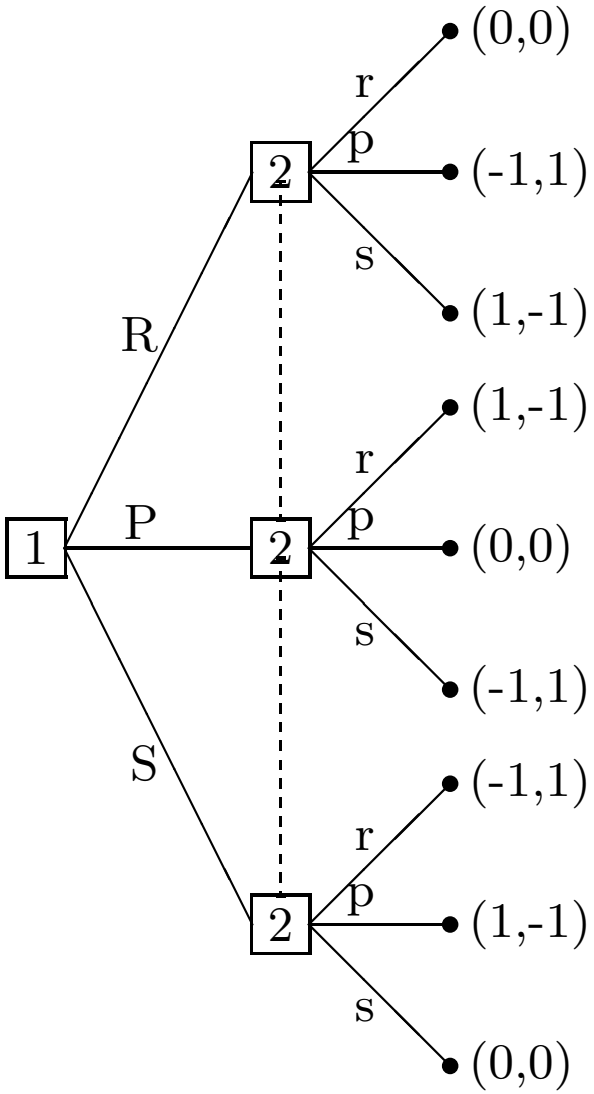
If the Challenger can't observe the Defender type:



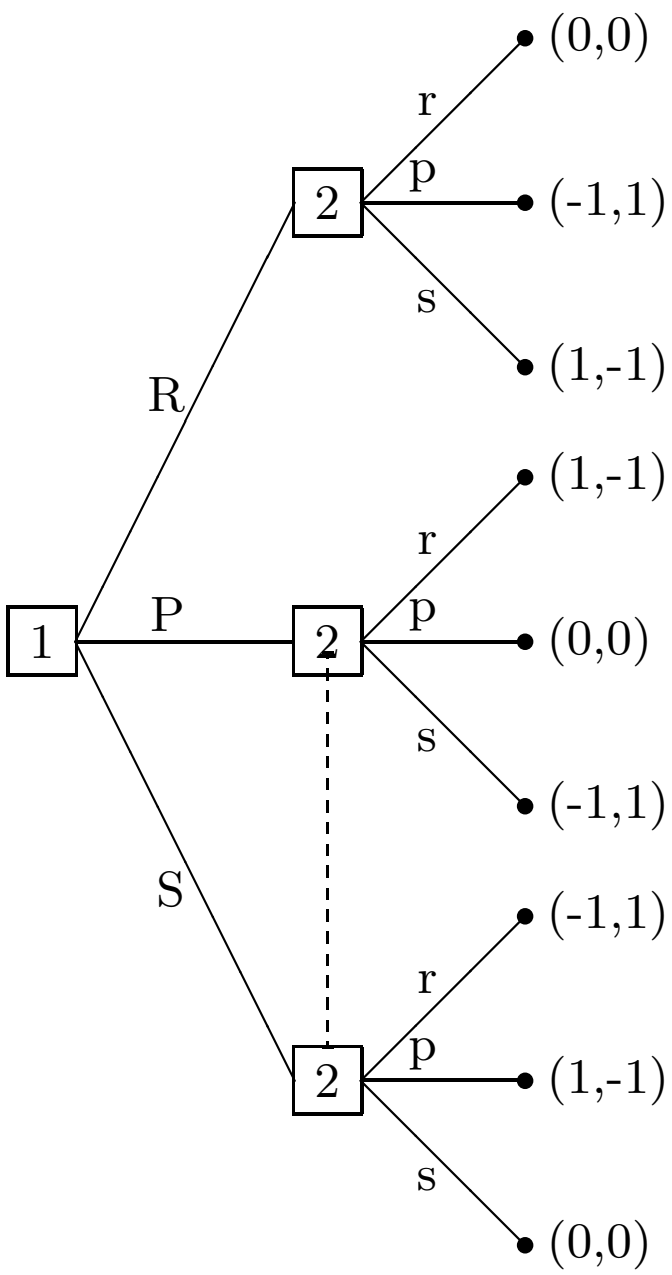
Example (Sequential Rock, Paper, Scissors):



Simultaneous Version: **Information Sets**.

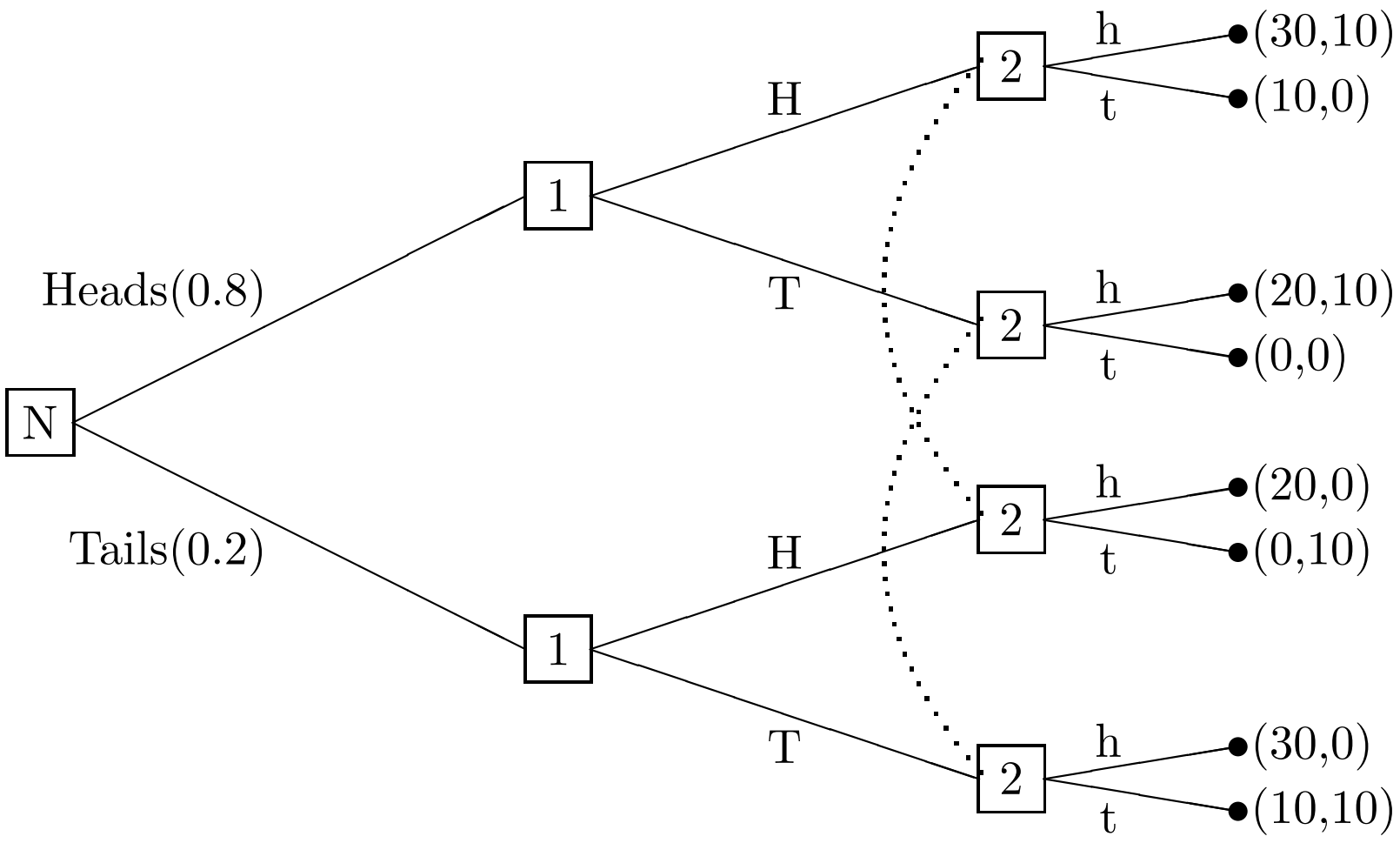


Indeed with the use of *information sets* we can represent all kinds of situations. For example suppose Player 2 can observe if Player 1 choose Rock, but cannot observe if the choice is Paper or Scissors if Player 1 choose Paper or Scissors:


1
R
P
S
2
2
2
r
p
s
r
p
s
r
p
s
(0,0)
(-1,1)
(1,-1)
(1,-1)
(0,0)
(-1,1)
(-1,1)
(1,-1)
(0,0)

Example (The Truth Game): There is an uneven coin which comes up Heads 80% of the time. There are two players.

1. Player 1 flips the coin and observes the result.
2. Player 1 announces Heads (H) or Tails (T).
3. Player 2 hears Player 1's announcement but cannot observe the result of the actual coin flip. Player 2 announces heads (h) or tails (t).
4. Payoffs are as follows:
   - Player 2 receives \$10 if his answer is true and \$0 otherwise.
   - Player 1 receives \$20 if Player 2 announces heads, and an additional \$10 if she tells the truth about the coin flip.


N
Heads(0.8)
Tails(0.2)
1
1
H
T
H
T
2
2
2
2
h
t
h
t
h
t
h
t
(30,10)
(10,0)
(20,10)
(0,0)
(20,0)
(0,10)
(30,0)
(10,10)

Now we are ready to give the formal definitions.

An **n-person extensive form game** consists of:

1. A finite **game-tree** composed of nodes and branches (where each node is either a move or an endpoint).
2. A division of the nodes over players, chance (nature), and endpoints.
3. Probability distribution for each chance move.
4. A division of each player's nodes into **information sets**.
5. A set of outcomes and an outcome to each endpoint.
6. A payoff (or utility) function for each player over all outcomes.

All this is **common knowledge** to all players.

- A **game-tree** consists of a series of **nodes** linked in a sequence. Each node that is not an endpoint (i.e. each **non-terminal node**) has a number of **branches** which lead to other nodes. Here
  - Non-terminal nodes represent decisions,
  - branches represent actions, and
  - terminal nodes indicate that the game is over.

  Loops (i.e. cycles) are not allowed in game trees. This implies that there can be only one path between any pair of nodes. The complete sequence of moves that precedes a node is called the **history** of the game upto that point. Every node has a unique history which summarizes all prior moves.

- The non-terminal nodes are divided between the players and the nature (chance). These nodes indicate whose turn it is in the game.
- At each node assigned to the chance there is a probability distribution of its moves. Here the chance move allows us to include randomness into the game.

- Each player's nodes are further divided into **information sets**.
  - Information sets summarize a player's knowledge of prior moves when he/she must decide.
  - If there are more than one nodes in an information set, then the player knows that he/she needs to give a decision and also that he/she is in one of the nodes in the information set (but does not know which one).

  That is, information sets summarize when players make choices and what they know at that point. Information sets containing only one node are referred to as **singletons**.
- An **outcome** is assigned to each terminal node.
- Players have **preferences** over outcomes. We typically give the **payoffs** associated with each outcome.