Equity

- Every society has rules for sharing goods and burdens among its members.

  Example: Hunt share for some hunting tribes. (Position in the tribe and role in the hunt are essential in this case.)

- These **sharing rules**, which in many cases are elaborately defined, express a notion of **equity** in the division of jointly produced goods.

- Here *equitable* does not necessarilly mean *ethical* or *moral*, but rather what a society considers to be *appropriate* given the need, status, and contribution of its various members.

- As economies become more complex, notions of *common property* and *just shares* gradually evolve into a system of *private ownership.*

  Nonetheless there are several examples of common property in everyday life:

  - Spectrum of frequencies for radio and satellite communications
  - Deep ocean bed
  - New common property that results from formulation of new enterprices. (How to split the profits between two merging firms, cost allocation, etc.)
  - Duty serve in the Army

Allocation Problems and Rules

- An **allocation problem** arises whenever a bundle of resources, rights, burdens, or costs is temporarily held in common by a group of individuals and must be alloted to them individually.
- An **allocation (distribution)** is an assignment of the objects to specific individuals.

  Here objects may be goods or burdens
- *Example*:
  - Goods: Broadcasting frequency, profits, organs for transplantation
  - Burdens: Taxation, military duty, layoff

- Goods may come in many forms:
  - Homogenous and Divisible (Ex: money, water)
  - Heterogenous and Divisible (Ex: land, broadcasting freq., fishing grounds)
  - Homogenous and Indivisible (Ex: seats in legislature, exemptions from military duty)
  - Heterogenous and Indivisible (Ex: Kidneys for transplantations, jobs, dormitory rooms)
- An **allocation rule** is a method, process, or formula that allocates any given supply of goods among any group of claimants according to the observable characteristics of these claimants.

- *Example*:
  - An **apportionment rule:** How many seats should each party get in a legislature, for a given distribution of votes and total seats.
  - **Taxation rule**
  - **Kidney allocation rule**
  - **On-campus room allocation rule**
- The allocation rules that we study usually exhibit one of three broad concepts of equity:
  - **Parity:** Claimants are treated equally (either because they are equal, or there is no clear way to distinguish them).
  - **Proportionality:** Acknowledges the differences among the claimants and divides the good in proportion.
  - **Priority:** Asserts that the person with the greates claim gets the good.

Normative Theories of Justice

Three general theories of justice play prominent rule

- **Aristotle's Equity Principle:** A good should be divided in proportion to each claimant's contribution.

  Two limitations:

  1. Measuring individual contributions should be possible
  2. Goods must be divisible

- **Classical Utilitarianism:** Goods should be distributed so as to maximize the total welfare of the claimants.

  For this notion to make sense, utility must be understood as a measure of satisfaction that can be measured on a cardinal scale and added across individuals.

Limitations:

1. How to measure satisfaction?
2. How to compare different people's satisfaction?
3. May require imposing great harm on a few in order to confer a small benefit on many.

- **Rawls:** The least well-of group in the society should be made as well as possible. (Also known as the **maximin principle**.)

  Here well-of does not refer to subjective satisfaction but rather the means of instrument by which satisfaction is satisfied (such as economic income, opportunity, power, etc.)

  Limitation:

  – Is it just to impose serious inconveniences on everyone in order to raise the opportunities of the least fortunate?

The conceptual difficulties posed by *utilitarian* and *Rawlsian* principles have led some economists to adopt an entirely different approach to distributive justice.

Envy-free Principle: An allocation should be such that no one prefers another's portion to his/her own.

- It does not require *interpersonal utility comparisons.*
- Initially *envy-freeness* is proposed in a very strong form: No one in the society should want to change places with another.

  Here "place" means occupation, income, health, size, etc.

  This is an attractive principle but it is impossible to achieve.

- A more pragmatic formulation: Do not require a society in general to be envy-free; only require that no person prefers another's portion of a *particular allocation of goods.*

  Example: In estate distribution among heirs, no heir should envy anothers portion of the property. They may, on the other hand, envy other things.

- This concept makes sense provided that the parties have equal claims on the goods, and the goods are divisible.

  Example:

  - Money (how to divide is obvious in this case)
  - A piece of land
  - Time on tennis courts

- The principle is more interesting if property consists of different kinds of goods.

Example: A & B need to share $200,000 and 300 acres of land. A values the land at $1000 an acre and B values the land at $500 an acre.

If A gets all the land and B gets all the money, that is envy-free. (It is also efficient since they have no desire to trade).

Typically there are many allocations that are envy-free and efficient. (For example we can give some additional money to A).

- One can find several allocations that are envy-free but are all of them equally fair?

- One way to resolve this problem is to use an allocation process that is perceived to be fair by both parties.
- *Example*: A coin toss determines a divider who divides the property into two parts and the other person chooses among the two pieces.
  - The final allocation is envy-free.
  - Here the divider can manipulate if he/she knows the preferences of the chooser. For example in the earlier example A could split the estate into
    1. \$175,001 and no land
    2. \$24,999 and all the land

    This is the best envy-free allocation for A!

Equity and Priority

- Let's consider the problem of allocating an indivisible good when each consumer can get at most one unit of the good which is in short supply.

  Example:

  - Which patient should receive the organ for transplantation?
  - Which soldier should be allowed to go home first?
  - Who gets the corner office?

There are various methods to solve such problems:

- **Forced Equality:** Give the good to no one.

  Example:

  - Works of art may deteriote because of exposure to many visitors. Rather than admit some, admit no-one.
  - Compulsory military service.

- **Lottories:** Give everyone an equal chance of getting the good or bearing the burden.

  Example:

  - During the last part of the Vietnam War US military called up soldiers according to their birth dates, which are prioritized by a random draw.
  - On campus housing for graduate students.

- **Rotation:** In some cases it may be more appealing to divide an indivisible good by taking turns.

  Example:

  - Who should wash the dishes?
  - Children are often time-shared between divorced parents.

- **Compensation:** Compensate those who do not get the good.

  Example:

  - The heir who gets the summer house could pay the siblings their share of its assesed value.
  - The town that is selected for the hazardous waste dump can be compensated by the towns that are not selected.

- **Queuing:** Give the goods to those who are first in line.

  Example:

  - Lining up for free tickets for a public concert.
  - Who should enroll to the special high school in Ann Arbor?

- **Priority Lists:** A more general form of a waiting list is the priority list, in which claimants are ranked according to some measure of need, contribution, seniority, or typically a combination of all.

  Example:

  - Admission to public housing (depends on financial need, family size, time on line, etc.)
  - Organ allocation for transplantation.

- Priority lists are probably the most widely used of any of the above methods. They are
  - simple,
  - they allocate the good itself (instead of a lottery, etc) and they make the basis of the allocation explicit.

  We have two case studies.

The Demobilization of US Soldiers at the End of WW II

- Beginning in 1943 the army surveyed thousands of soldiers to identify the factors they considered most important in the decision about whom to let out first.

  4 main factors:

  - Length of time in the army
  - Age
  - Amount of Overseas Service
  - Number of Dependants

- They have been asked o compare these in a pairwise manner.
- About 90% of the responses were internally consistent.

- Results

	% of Responses in Which the Factor was Ranked			
	1st	2nd	3rd	4th
Overseas Service (O)	54	39	6	1
Dependents (D)	38	26	25	11
Time in Army (L)	5	26	43	26
Age (A)	3	9	26	62

- Based on these results: O > D > L > A
- Soldiers are asked to add an additional criteria if they want. They added "Exposure to Combat."
- The Army also wanted to determine the importance weights. In a follow up survey in August 1944, the soldiers are asked questions such as:

Here are 3 men of the same age, all overseas the same length of time. Check the one you would want to have let out first.

a- Single, through 2 campaigns of combat

b- Married, no children, one campaign of combat

c- Married, 2 children, not in combat

- Based on these surveys, the army command adopted the following point system.

Time in Army:	1 point/month
Overseas Service:	1 point/month
Combat:	5 points per campaign star
	or combat decoration
Dependents:	12 point per child under 18
	up to 3 children

Soldiers with 85 points or more were demobilized first.

The Old Point System for Allocating Kidneys in US

- *Policy*: Organs offered for transplantation are viewed as natural resources.
- A survey is conducted and this lead to the adoption of three broad criteria of entitlement:
  1. **Efficacy:** The likelihood that the transplant will be succesfull.
  2. **Need:** The lack of alternatives such as dialysis.
  3. **Disadvantage:** Patients who are difficult to match should be given a handicap.
- Patients are sorted according to the types of kidneys for which they are medically eligible: Kidneys should be in right size, right blood type, right tissue type, and the recipient should not have cytotomic antibody formations against the kidney.

- *The point system*:
  - Two points for each of possible antigen matches. (The more there is antigen match, the higher the success rate.)
  - A bonus of upto 6 points if the logistics of gettttting the kidney is favorable.
  - A bonus of upto 10 points if all available dialysis sites is used up. (10 points for medical urgency.)
  - Some patients are disadvantaged because they are highly sentized; that is they have antibodies against a high proportion of the rest of the population. 1 point for each 10% of the general population against which they have antibodies.
  - 10 points minus ten times the fraction of patients who have been waiting longer.

Patient	Months	Antigen	Sens.	Log.	Urgency
A	5	2	10	0	0
B	4.5	2	20	0	0
C	4	0	0	5	0
D	2	3	60	0	0
E	1	6	90	0	0

Patient	Months	Antigen	Sens.	Log.	Urgency	Total
A	10	4	1	0	0	15
B	8	4	2	0	0	14
C	6	0	0	5	0	11
D	4	6	6	0	0	16
E	2	12	9	0	0	23

In this example first table gives the patient characteristics and the second one gives the point values. Suppose there are two kidneys. They are awarded to patients D and E.

Next suppose the kidneys arrive one at a time, and patient E has already left with the first kidney. Let's obtain the points for the remaining patients:

Patient	Months	Antigen	Sens.	Log.	Urgency	Total
A	10	4	1	0	0	15
B	7.5	4	2	0	0	13.5
C	5	0	0	5	0	10
D	2.5	6	6	0	0	14.5

In this case Patient A receives the second kidney!

- In other words, when there are 2 kidneys patient D is favored against patient A whereas when there is one less patient and one less kidney patient A is favored against patient D!
- A rule is **pairwise consistent** if the decision between two types are always made in the same way independently of the other claimants and how much they receive.

  As the above example shows, the point system that is used in kidney allocation in US is not pairwise consistent.

How Should Priority be Determined?

- In both applications there was a survey. One important issue is *aggregating individual rankings* into a *social ranking*.

Example:

Patient	Efficacy	Urgency	Time Waiting
A	10	2	1.5
B	5	1	3
C	20	5	0.5

- Efficiacy measure is the expected years of life if the operation is performed,
- Urgency measure is the expected years of life if the operation is not performed, and
- Time waiting is in years.

# of Committee Members with Ranking

13	10	6	13	18
A	A	B	B	C
C	B	A	C	B
B	C	C	A	A

- **Borda's Rule:** Borda score of each alternative is the total number of alternatives that are ranked below, summed over all individual opinions.

  Borda's rule orders the alternatives according to their Borda score.

- **Example continued:**

  $A = 13 \times 2 + 10 \times 2 + 6 \times 1 + 13 \times 0 + 18 \times 0 = 52$

  $B = 13 \times 0 + 10 \times 1 + 6 \times 2 + 13 \times 2 + 18 \times 1 = 66$

  $C = 13 \times 1 + 10 \times 0 + 6 \times 0 + 13 \times 1 + 18 \times 2 = 62$

  Therefore the Borda ranking is BCA

- Borda rule is simple and natural but it has a serious drawback: In the example C obtains a strict majority over A and B and yet it ranks second.

  Condercet suggested that based on this observation it should be ranked first.

- **Majority Alternative:** A majority alternative is an alternative that would receive a strict majority of the votes when compared pairwise with every alternative.

  Clearly the Borda rule does not rank the the majority alternative first.

- Condorcet suggested to choose the ranking that is supported by the maximum number of pairwise votes.

- **Example continued:** The ranking BCA contains 3 propositions:

  B has priority over C: supported by 29 votes

  C has priority over A: supported by 31 votes

  B has priority over A: supported by 37 votes

  Therefore the Condercet score of BCA is 29+31+37=97.

  Similarly the Condercet score of CBA is 99. This is the ranking with highest Condercet score.

- **Condercet's criterion:** Given a set of types and a ranking of these types by each member of a group, a Condercet ranking is the one in which the pairwise assertions of priority is supported by the maximum number of individuals.
- If there is a majority alternative, Condercet's criterion ranks it first.
- Note that in many situations there is no majority alternative.

Example:

6 voters	5 voters	2 voters
A	B	C
B	C	A
C	A	B

Here there is no majority alternative. Indeed A beats B, B beats C, and C beats A. This is known as **Condercet Paradox** (or **paradox of voting**).

- Examples like this show that there is no obvious way to extend simple majority rule to situations with more than 2 alternatives. Consider the following three requirements:
- **Unanimity:** If all voters rank some alternative A above another alternative B, then A is ranked above B in the concensus ranking.
- **Non-dictatorship:** The concensus ranking is not dictated by the same individual in all situations.
- **Independence of irrelevant alternatives:** The relative rank of each pair of alternatives in the concensus order depends only on the individual opinions regarding that pair.
- **Arrow's Impossibility Theorem:** There is no method of aggregating individual rankings into a single concensus ranking that meets unanimity, non-dictatorship, and independence of irrelevant alternatives.