EC 308 Sample Exam Questions

Sample Midterm 1

Multiple Choice Questions

1. In the following game

		Player 2		
		l	m	r
Player 1	U	2,0	3,1	0,0
	M	1,1	2,2	1,2
	D	3,2	2,2	2,1

   (a) D dominates M for player 1 and therefore D is a dominant strategy.

   (b) There is no dominant strategy for either players.

   (c) M is a dominated strategy for player 1 and m is a dominant strategy for player 2.

   (d) (U;m) and (D;l) are dominant strategies.

   (e) None of the above.

2. In the following game

		Player 2		
		l	m	r
Player 1	U	2,2	1,2	2,1
	M	1,3	2,3	1,4
	D	1,1	2,2	2,1

   (a) there is no Nash equilibrium.

   (b) D is a dominant strategy for player 1 and m is a dominant strategy for player 2.

   (c) M is a dominant strategy for player 1 and l is a dominant strategy for player 2.

   (d) there are no dominated strategies for either players.

   (e) the only strategy pair that survives iterative elimination of dominated strategies is (D,m).

3. In the following game

		Player 2	
		L	R
Player 1	U	2,1	3,0
	M	2,1	1,2
	D	2,2	1,2

(a) (U;L) and (D;L) are the only Nash equilibria.

(b) (U;R) and (D;L) are the only Nash equilibria.

(c) (U;R) is the only Nash equilibrium.

(d) (D;L) is the only Nash equilibrium.

(e) None of the above.

4. In the following 3 person simultaneous game player 1 chooses the row, player 2 chooses the column, and player 3 chooses the box.

		Player 3: A			Player 3: B	
		Player 2			Player 2	
		L	R		L	R
Player 1	U	2,0,2	1,1,1	U	0,2,2	1,1,1
	D	1,1,1	0,2,2	D	1,1,1	2,0,2

In this game

(a) (D,L,A) and (D,L,B) are the only Nash equilibria.

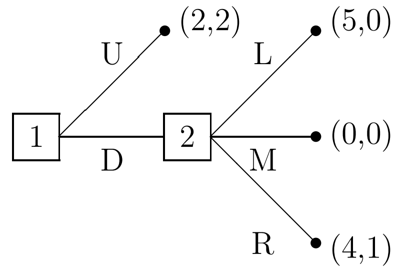
(b) (U,R,A) and (D,L,B) are the only Nash equilibria.

(c) (D,L,A) and (U,R,B) are the only Nash equilibria.

(d) (U,R,A) is the only Nash equilibrium.

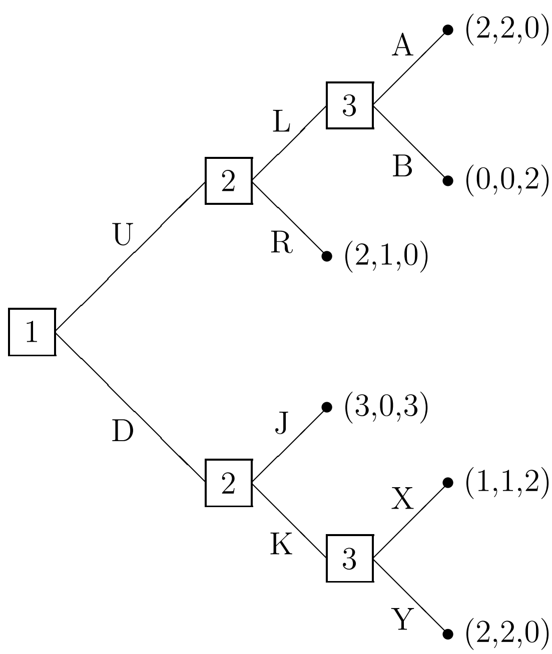
(e) (U,R,B) is the only Nash equilibrium.

5. Consider the following two person sequential game. In this game



(a) (U,M) and (D,R) are the only Nash equilibria.
(b) (D,R) and (D,L) are the only Nash equilibria.
(c) (U,M) is the only Nash equilibrium.
(d) (D,R) is the only Nash equilibrium.
(e) (D,L) is the only Nash equilibrium.

6. The following sequential game has a unique subgame perfect Nash equilibria.



It is

(a) (U,R)
(b) (U,R,B)
(c) (D,J)
(d) (UR,B,KX)
(e) (U,RK,BX)

7. The following game can be solved by iteratively eliminating dominated strategies. The order of the elimination is

		Player 2		
		l	m	r
Player 1	U	-3,1	-1,4	-2,0
	M	7,3	-1,0	2,5
	D	0,4	4,-1	0,2

(a) first D (for player 1), second m (for player 2), third U (for player 1), last r (for player 2) and the remaining strategy pair is (M;l) yielding (7,3).

(b) first D (for player 1), second U (for player 1), third m (for player 2), last r (for player 2) and the remaining strategy pair is (M;l) yielding (7,3).

(c) first U (for player 1), second l (for player 2), third D (for player 1), last m (for player 2) and the remaining strategy pair is (M;r) yielding (2,5).

(d) first U (for player 1), second m (for player 2), third D (for player 1), last l (for player 2) and the remaining strategy pair is (M;r) yielding (2,5).

(e) None of the above.

8. In an extensive form game two nodes are in the same information set whenever

(a) they belong to different players and players cannot differentiate between them.

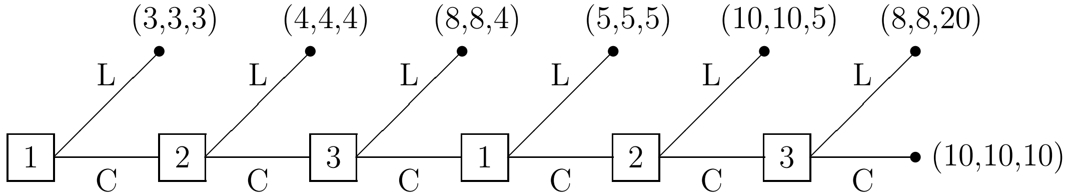
(b) they belong to the same player and that player cannot differentiate between them.

(c) they belong to different players and players can differentiate between them.

(d) they belong to the same player and that player can differentiate between them.

(e) None of the above.

9. If we solve the following game with backwards induction the resulting payoff is



(a) (3,3,3)
(b) (4,4,4)
(c) (8,8,4)
(d) (10,10,5)
(e) (8,8,20)

Problems

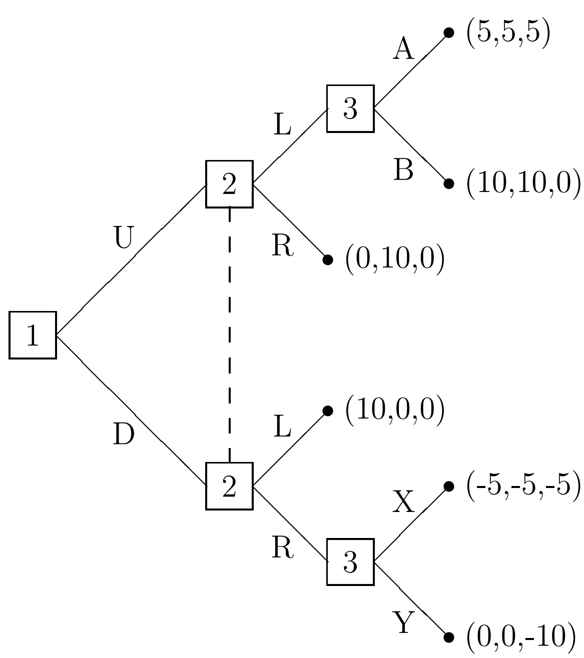
10. Bob and Sue will eventually play the following game:

		Sue L	Sue R
Bob	U	3,1	0,0
	D	0,0	1,3

In a preliminary stage Bob has already proposed Sue to pay 1 units of the payoff so that he can move first.

Represent the rest of this situation as an extensive form game: First Sue decides to accept (A) or reject (R) this offer. If she rejects, they simultaneously play the game. If she accepts, Bob moves first and observing his move Sue moves next. (In the second case do not forget to adjust the payoffs.)

11. Identify the subgames of the following game and find all it's subgame perfect Nash equilibria.



Sample Midterm 1

Multiple Choice Questions

1. In the following game

Player 2

		l	m	r
	U	1,2	3,1	2,2
Player 1	M	0,0	1,1	1,1
	D	1,1	4,0	2,1

   (a) There are four Nash equilibria.
   (b) (D,r) is a dominant strategy equilibrium.
   (c) M is a dominated strategy for Player 1.
   (d) All of the above.
   (e) None of the above.

2. In the following game

Player 2

		L	R
	U	2,2	1,4
Player 1	M	0,5	0,0
	D	1,1	2,2

   (a) There is no Nash equilibrium.
   (b) (U,L) is a Nash equilibrium.
   (c) Only the strategy profile (D,R) survives iterated elimination of dominated strategies.
   (d) No player has a dominated strategy.
   (e) U is a dominant strategy.

3. The following game can be solved by iteratively eliminating dominated strategies. The strategy profile that survives the elimination is:

Player 2

		L	M	R
	A	3,1	2,2	2,0
Player 1	B	5,2	1,3	1,4
	C	2,5	1,2	2,3
	D	4,1	3,2	1,2

(a) (C,L)

(b) (D,L)

(c) (B,R)

(d) (D,M)

(e) (C,R)

4. In the following game

P3

A

P2

		L	R
	U	1,1,1	1,2,4
P1	M	2,1,0	3,1,0
	D	3,1,1	3,2,0

B

P2

	L	R
U	2,2,2	1,2,1
M	3,0,0	2,1,0
D	1,1,1	4,1,1

(a) (D,R,A) and (U,L,B) are the only Nash equilibria.

(b) (M,R,A) and (D,R,B) are the only Nash equilibria.

(c) (D,R,A) and (D,R,B) are the only Nash equilibria.

(d) (M,R,A) and (U,L,B) are the only Nash equilibria.

(e) (D,R,A) and (M,R,B) are the only Nash equilibria.

5. In the following game

		Player 2				
		V	W	X	Y	Z
	A	2,2	1,6	1,3	2,1	5,5
	B	6,1	3,4	4,4	1,3	1,6
Player 1	C	4,1	3,2	3,0	1,2	4,1
	D	1,2	2,3	1,1	2,3	2,0
	E	5,5	2,4	1,3	0,2	2,2

(a) (E,V) and (A,Z) are the only Nash equilibria.

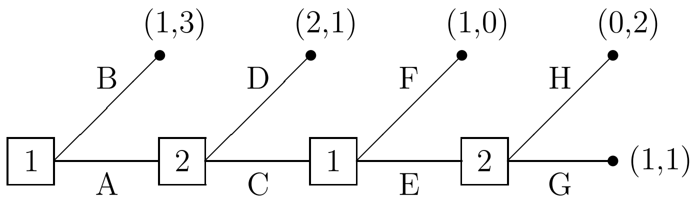
(b) (C,W) and (D,Y) are the only Nash equilibria.

(c) (B,V), (A,Z) and (B,X) are the only Nash equilibria.

(d) (A,V) and (E,Z) are the only Nash equilibria.

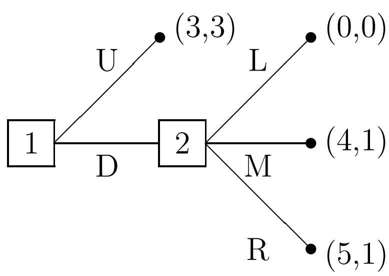
(e) There is no Nash equilibrium.

6. If we solve the following game with backwards induction the resulting payoff is:



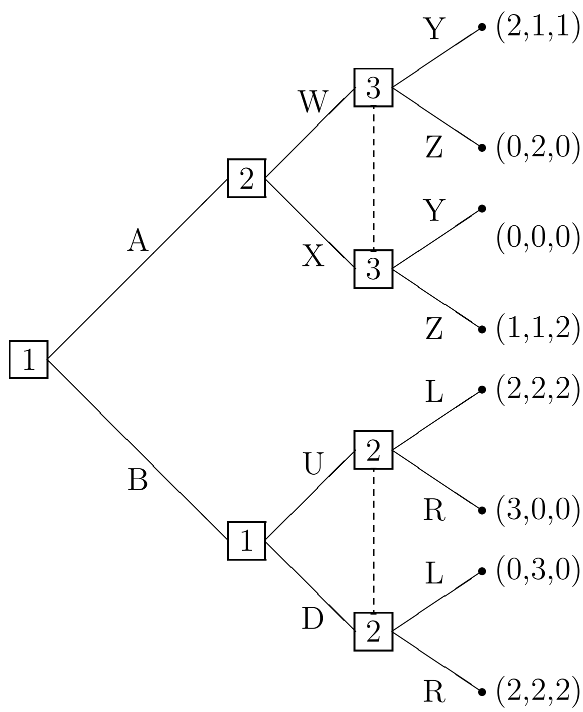
(a) (1,3)
(b) (2,1)
(c) (1,0)
(d) (0,2)
(e) (1,1)

7. In the following extensive form game one of the Nash equilibria is NOT a subgame perfect Nash equilibrium. Which one?



(a) (U,L)
(b) (U,M)
(c) (U,R)
(d) (D,M)
(e) (D,R)

8. Consider the equivalent strategic form game of the following extensive form game.



In this game

(a) Player 1 has 4 strategies, Player 2 has 4 strategies, and Player 3 has 2 strategies.

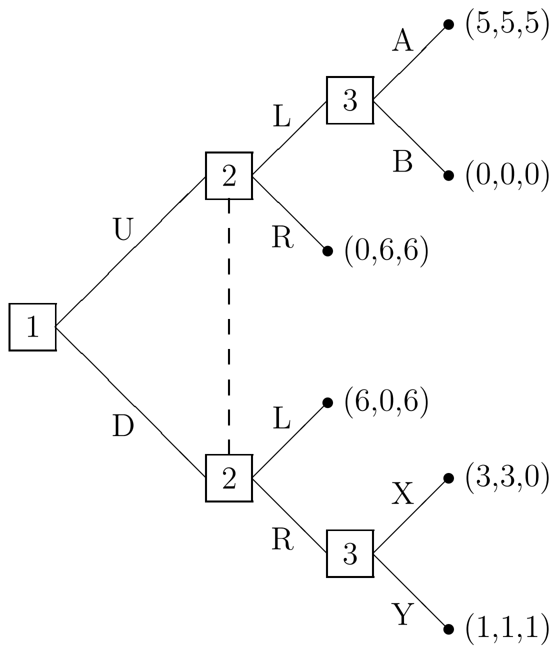
(b) Player 1 has 4 strategies, Player 2 has 6 strategies, and Player 3 has 4 strategies.

(c) Player 1 has 4 strategies, Player 2 has 8 strategies, and Player 3 has 4 strategies.

(d) Player 1 has 2 strategies, Player 2 has 2 strategies, and Player 3 has 1 strategy.

(e) Player 1 has 2 strategies, Player 2 has 6 strategies, and Player 3 has 4 strategies.

9. The following game has only one subgame perfect Nash equilibrium.



It is

(a) (U,R,AY)
(b) (D,L,AY)
(c) (D,R,AY)
(d) (U,L,AY)
(e) (U,L,A)

Problems

10. Represent the following scenario as an extensive form game.

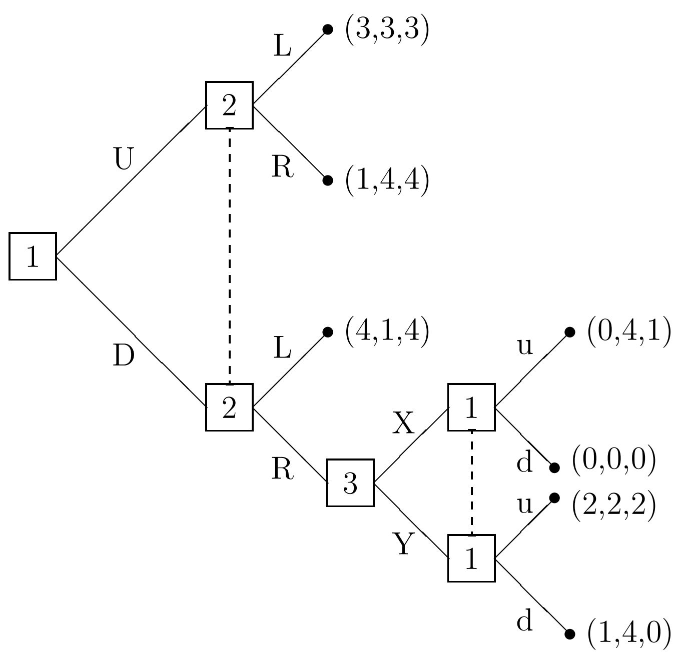
Three legislators are trying to decide whether they should give themselves a pay raise of \$b. Legislator 1 does not have a vote but she determines the format the other two legislators vote. The sequence of events is as follows: First legislator 1 decides whether the vote should be simultaneous (action A) or sequential (action B). Her choice is observed by legislators 2 and 3.

- If legislator 1 chooses action A, then legislators 2 and 3 simultaneously choose between Yes (action Y) and No (action N).
- If legislator 1 chooses action B, then legislator 2 chooses between Yes (action y) and No (action n) and after observing his vote legislator 3 chooses between Yes (action y) and No (action n).

The payoffs are determined as follows:

- There is a pay raise only if both legislators 2 and 3 vote Yes. The benefit from a pay raise is \$b for all three legislators.
- Those legislators who vote Yes pay a cost of \$c whether there is a pay raise or not.

11. Consider the following 3 person extensive form game:



(a) Find the equivalent strategic game of this extensive form game.

(b) Find all the subgame perfect Nash equilibria of this extensive form game. (Please give equilibrium strategies as well as payoffs.)

Sample Midterm 2

Multiple Choice Questions

1. In prisoner's dilemma

   (a) cooperation cannot be sustained if the game is played only once.

   (b) cooperation can be sustained if the game is played repeatedly.

   (c) cooperation may not be sustained if the game is played finite number of times and both players know when the final game will be played.

   (d) All of the above.

   (e) None of the above.

2. The mixed strategy Nash equilibrium of the following game is

		Player 2	
		L	R
Player 1	U	2,2	3,1
	D	3,-1	0,0

   (a) U with 1/2 probability and D with 1/2 probability for player 1; L with 3/4 probability and R with 1/4 probability for player 2.

   (b) U with 1/2 probability and D with 1/2 probability for player 1; L with 1/4 probability and R with 3/4 probability for player 2.

   (c) U with 3/4 probability and D with 1/4 probability for player 1; L with 1/2 probability and R with 1/2 probability for player 2.

   (d) U with 1/4 probability and D with 3/4 probability for player 1; L with 1/2 probability and R with 1/2 probability for player 2.

   (e) None of the above.

3. There are three alternatives A,B,C. Voters are first voting between A and B and then between the winner and C. At each step the winner is the alternative that gets the majority of the votes and the final winner is the winner of the second vote. Here

   (a) the median voter theorem applies.

   (b) the outcome may be different if the order of the voting differs.

   (c) voters cannot benefit by voting strategically.

   (d) the final winner should be the top choice of at least half of the voters.

   (e) None of the above.

4. Consider the following voting problem: There are four alternatives A, B, C, D, and three voters with the following rankings:

Voter 1	Voter 2	Voter 3
A	B	C
B	C	D
C	D	A
D	A	B

The order of the voting is such that, first A competes against B, next C competes against D, and finally the two winners compete to determine the final outcome. (At each step the winner is determined by the majority rule.) In this game

(a) the sophisticated voting outcome is C.

(b) the sophisticated voting outcome is A.

(c) the sophisticated voting outcome is B.

(d) the sincere voting outcome is A.

(e) the sincere voting outcome is B.

Short Answer Questions

5. What is the median voter theorem?
6. When can cooperation be sustained in a finitely repeated prisoner's dilemma?
7. If a goal keeper's strong side is his left side, then he should always move (or "jump") towards left during penalty shots.
8. What is the difference between sincere and sophisticated voting?
9. True or False? Explain: The ratio of drivers to public transportation users is at the socially efficient level as it is determined by the market mechanism.

Problems

10. The success of a project depends on the effort level of a worker. In case of success the revenues will be \$ 300,000 whereas in case of no-success the revenues will be \$ 0. The worker can provide a low effort level or a high effort level. The effort level cannot be observed by the principal. The worker requires an expected salary of \$ 50,000 to provide the low effort level and an expected salary of \$ 60,000 to provide the high effort level. The success probabilities for the project are 75 % in case of high effort level and 25 % in case of low effort level. What should be the minimum bonus (that is avarded in addition to the base salary in case of success) and the base salary for the worker so that the worker has the incentives to provide the high effort level?

11. Consider the following voting problem: There are four alternatives A, B, C, D, and five sophisticated voters with the following rankings:

Voter 1	Voter 2	Voter 3	Voter 4	Voter 5
A	B	C	D	B
B	C	D	A	D
C	D	A	B	A
D	A	B	C	C

Which agenda does voter 1 prefer? (1) A versus B; winner versus C; winner versus D; or (2) A versus B; C versus D; winners versus each other; or (3) C versus D; winner versus A; winner versus B. Explain.

12. Consider the following voting problem: There are four alternatives A, B, C, D, and five voters with the following rankings:

Voter 1	Voter 2	Voter 3	Voter 4	Voter 5
A	B	C	D	B
B	C	D	A	D
C	D	A	B	A
D	A	B	C	C

The order of the voting is such that, first A competes against B, next the winner competes with C, and finally the winner competes by D to determine the final outcome. (At each step the winner is determined by the majority rule.) Find the final outcome if voters 3 and 4 are sophisticated voters (and they know that only they are sophisticated) and the rest of the voters are sincere voters. (Please show your work.)

Sample Midterm 2

Multiple Choice Questions

1. In the following game the payoffs to the Receiver is the probability of saving and the payoffs to the Server is the probability of scoring.

		Server (Player 2) forehands	Server (Player 2) backhands
Receiver (Player 1)	Forehands	80,20	0,100
	Backhands	30,70	50,50

   We know that the Receiver will mix between her Forehands and Backhands in order not to be exploited by the Server. What is the best mix for the Receiver?

   (a) Forehands with 0.1 probability and Backhands with 0.9 probability.

   (b) Forehands with 0.2 probability and Backhands with 0.8 probability.

   (c) Forehands with 0.3 probability and Backhands with 0.7 probability.

   (d) Forehands with 0.4 probability and Backhands with 0.6 probability.

   (e) Forehands with 0.5 probability and Backhands with 0.5 probability.

2. The following game has a mixed strategy Nash equilibrium.

Player 1 \ Player 2	W	X	Y	Z
A	6,1	0,2	3,2	2,3
B	4,2	1,3	6,1	3,1
C	3,1	2,2	4,3	2,1
D	1,5	1,2	3,3	1,0

   It is:

   (a) B with 1/3 probability and C with 2/3 probability for Player 1 and X with 2/3 probability and Y with 1/3 probability for Player 2.

   (b) A with 2/3 probability and B with 1/3 probability for Player 1 and W with 2/3 probability and X with 1/3 probability for Player 2.

   (c) A with 1/3 probability and B with 2/3 probability for Player 1 and W with 2/3 probability and X with 1/3 probability for Player 2.

   (d) B with 2/3 probability and C with 1/3 probability for Player 1 and X with 1/3 probability and Y with 2/3 probability for Player 2.

   (e) A with 2/3 probability and B with 1/3 probability for Player 1 and X with 1/3 probability and Y with 2/3 probability for Player 2.

3. One of the following statements is false. Which one?

   (a) Truthtelling is a dominant strategy under the majority rule when there are two alternatives.

   (b) Suppose that the voters are voting on a single-dimensional issue. Each voter has a favorite point on the spectrum and the closer the current policy is towards their favorite point the better they are. Suppose that the outcome is determined by the median voter's position. Under these conditions truthtelling is a dominant strategy for each voter.

   (c) Agenda manipulation is the process of orginizing the order of pairwise competitions in voting trees in order to assure favorible outcomes.

   (d) Truthtelling is not a dominant strategy under the plurality rule.

   (e) Pairwise competitions in voting trees always yield Pareto efficient outcomes.

4. There are two means of transportation for commuting between two cities A and B:

   - Driving: It takes 30 minutes if there is no one else driving and there is a delay of 5 minutes for every 10,000 drivers.
   - Train: 45 minutes no matter what.

   There are 80,000 commuters and they only care for minimizing their own commuting time.

   Which of the following statements is false?

   (a) At equilibrium there are 50,000 commuters using the train.

   (b) The equilibrium outcome is inefficient.

   (c) There are three equilibria: (1) Everybody drives, (2) everybody uses the train, (3) a mixed equilibrium.

   (d) It may be possible to avoid inefficiencies with some regulations.

   (e) None of the above.

5. You are a member of a three person committee that must choose one outcome from the list (A,B,C,D). Suppose the rankings (from best to worst) are:

You	Voter 2	Voter 3
B	C	D
A	A	C
D	D	B
C	B	A

   Which of the following procedures would you prefer to see implemented if everyone is sophisticated?

(a) First A vs. B, next C vs. D, and finally the two winners vs. each other.
(b) First A vs. B, next the winner vs. C, and finally the survivor vs. D.
(c) First A vs. D, next B vs. C, and finally the two winners vs. each other.
(d) First D vs. C, next the winner vs. B, and finally the survivor vs. A.
(e) Indifferent between all procedures.

6. There are three voters and four alternatives. Voter preferences (from best to worst) are as follows:

Voter 1	Voter 2	Voter 3
A	B	C
B	C	A
C	A	B

The voting procedure is as follows: First A competes with B and then the winner competes with C.

One of the following statements is true. Which one?

(a) The sincere voting outcome is A.
(b) The sophisticated voting outcome is A.
(c) The sophisticated voting outcome is C.
(d) The outcome is B if voter 1 is sophisticated and the others are sincere.
(e) The outcome is C if voter 1 is sophisticated and the others are sincere.

7. You are one of several drivers in a city. Each driver's utility from driving is

$$u = 50 - t^2 + 6t - 2t_o$$

where t is the number of hours in a day of own driving and t_o is the number of hours others drive on average.

Suppose there are no regulations. How many hours will each driver drive per day and is this efficient?

(a) 2 hours and efficient
(b) 2 hours and inefficient
(c) 3 hours and inefficient
(d) 4 hours and efficient
(e) 4 hours and inefficient

8. The success of a project depends on the effort level of a worker. In case of success the revenues will be \$300,000 whereas in case of no-success the revenues will be \$0. The worker can provide a low effort level or a high effort level. The effort level cannot be observed by the principal. The worker requires an expected salary of \$50,000 to provide the low effort level and an expected salary of \$60,000 to provide the high effort level. The success probabilities for the project are 70% in case of high effort level and 50% in case of low effort level. What should the base salary be for the worker in an optimal contract?

   (a) \$15,000

   (b) \$25,000

   (c) \$30,000

   (d) \$40,000

   (e) \$50,000

Short Answer Questions

9. True or False? Explain: Consider a finitely repeated Prisoner's Dilemma game where the game is played T times (where T is a finite integer). It is not possible to sustain cooperation in this game if both players know that the game will end in finite periods; but neither knows what T is.

10. There is a common feature of the pivotal mechanism and the second price auction. What is it?

11. Consider the following game:

		Server (Player 2) forehands	Server (Player 2) backhands
Receiver (Player 1)	Forehands	100,0	0,100
	Backhands	100-Π,Π	100,0

   where $0 \leq \Pi \leq 100$. In this game forehands is the server's weaker side: While he can score every backhands serve if the Receiver moves Forehands, he can only score Π percent of the forehands serves if the Receiver moves Backhands.

   True or False? Explain: The better the server's forehands is (i.e the higher Π is) the more often the server aims forehands at equilibrium.

   Hint: You should find the mixed strategy Nash equilibrium in order to answer the question.

Problems

12. Consider the following voting problem: There are four alternatives A, B, C, D, and three voters with the following rankings:

Voter 1	Voter 2	Voter 3
B	D	A
D	C	C
A	B	B
C	A	D

The order of the voting is such that, first A competes against B, next C competes against D, and finally the two winners compete to determine the final outcome. (At each step the winner is determined by the majority rule.) Find the final outcome if voters 2 and 3 are sophisticated voters and voter 1 is a sincere voter. Clearly indicate the vote of each voter in the first two round of votes (i.e. the vote between A/B and the two votes between C/D).

13. There is a single asset which will be sold through an auction. The rules of the auction is as follows:

- Individuals submit sealed bids.
- The asset goes to the highest bidder and this bidder pays the second highest bid.

You are one of the individuals in the auction. Your valuation of the asset is \$1000 and your utility from buying the asset for a price of \$$p$ is $u = 1000 - p$. (In other words your objective is maximizing your net gain.) You are allowed to make any positive bid $B > 0$. Show that bidding \$1000 (i.e. bidding your valuation) is a dominant strategy for you.

Sample Final Exam

Multiple Choice Questions

1. One of the following statements is false. Which one?
   (a) The only zero-one allocation methods that are impartial and pairwise-consistent are priority methods.
   (b) The proportional rule is the only claims allocation rule that is impartial and collusion-proof.
   (c) None of Jefferson's, Webster's, Adams's, or Hill's methods suffer from the population paradox.
   (d) The Shapley value (in the context of claims allocation problems) is inconsistent.
   (e) The core is the only cost sharing rule that satisfies the dummy player property, symmetry, and additivity.

2. There are three alternatives A, B, C and seven voters with the following rankings:

Voter 1	Voter 2	Voter 3	Voter 4	Voter 5	Voter 6	Voter 7
A	A	C	C	A	B	B
B	B	B	B	C	A	C
C	C	A	A	B	C	A

   Here the Condorcet ranking (i.e. the ranking with the highest Condorcet score) is
   (a) ABC
   (b) ACB
   (c) BAC
   (d) BCA
   (e) CBA

3. Adams's apportionment method is
   (a) unbiased, it is consistent with the standard two-state solution, and it does not suffer from Alabama paradox.
   (b) biased, it is consistent with the standard two-state solution, and it does not suffer from Alabama paradox.
   (c) biased, it is not consistent with the standard two-state solution, and it does not suffer from Alabama paradox.

(d) unbiased, it is not consistent with the standard two-state solution, and it suffers from Alabama paradox.

(e) biased, it is not consistent with the standard two-state solution, and it suffers from Alabama paradox.

4. Consider the following cost sharing problem. There are three players A, B, C, with the following costs:

$$\begin{array}{lll} \mathrm{c(A)} = 7 & \mathrm{c(AB)} = 17 & \\ \mathrm{c(B)} = 8 & \mathrm{c(AC)} = 15 & \mathrm{c(ABC)} = 20 \\ \mathrm{c(C)} = 7 & \mathrm{c(BC)} = 13 & \end{array}$$

Consider a cost allocation that is in the core. Which of the following statements concerning such an allocation is false?

(a) B should not pay more than 8.

(b) B should not pay less than 5.

(c) A should pay exactly 7.

(d) C should not pay more than 7.

(e) C should not pay less than 5.

5. Consider the following claims problem: There are three individuals and a total of \$ 500 to divide between them. Claimant 1 claims \$ 50, claimant 2 claims \$ 250, and claimant 3 claims \$ 325. Find the solutions to this claims problem suggested by the Talmudic solution and the Maimonides' rule.

(a) Division by the Talmudic solution = (50, 225, 225) and division by the Maimonides' rule = (50, 225, 225).

(b) Division by the Talmudic solution = (25, 200, 275) and division by the Maimonides' rule = (50, 225, 225).

(c) Division by the Talmudic solution = (25, 200, 275) and division by the Maimonides' rule = (50, 200, 250).

(d) Division by the Talmudic solution = (50, 200, 250) and division by the Maimonides' rule = (25, 225, 250).

(e) Division by the Talmudic solution = (50, 200, 250) and division by the Maimonides' rule = (50, 200, 250).

Short Answer Questions

6. Arrow's Impossibility theorem states that there is no method for aggregating individual rankings into a single concensus ordering (or ranking) that meets three conditions. What are these conditions and what do they mean?

7. What is Alabama paradox? Which method(s) suffer from it?
8. Explain the normative theories of justice by Aristotle, Bentham, and Rawls. What are their weaknesses?
9. What is population paradox? Which method(s) suffer from it?
10. Explain the No-envy principle. Explain the consistency principle. (For the latter first explain in a general context and then for specific applications.)

Problems

11. Consider the following apportionment problem: There are a total of 11 seats to be allocated. There are four states with the following populations: State A (17), State B (22), State C (29), and State D (32). Find the allocations suggested by Hamilton's, Jefferson's, Webster's, Adam's, and Hill's methods. (Please show your work.)
12. There are three players A, B, C, with the following costs:

$$\begin{array}{lll} c(A) = 7 & c(A,B) = 17 & \\ c(B) = 8 & c(AC) = 15 & c(ABC) = 20 \\ c(C) = 7 & c(BC) = 13 & \end{array}$$

    Find the Shapley value of this cost sharing problem.

Sample Final Exam

Multiple Choice Questions

1. There are four alternatives A, B, C, D and five voters with the following rankings:

Voter 1	Voter 2	Voter 3	Voter 4	Voter 5
B	D	D	A	A
C	C	B	B	C
A	B	A	D	B
D	A	C	C	D

   One of the following rankings is the Condorcet ranking. Which one?

   (a) ABCD
   (b) ACBD
   (c) ABDC
   (d) BADC
   (e) CBAD

2. Which of the following claims allocation rules is collusion-proof?

   (a) Shapley value
   (b) Proportional rule
   (c) Talmudic solution
   (d) Maimonides' rule
   (e) None of the above

3. Consider the following cost sharing problem. There are three players A, B, C, with the following costs:

$$\begin{array}{lll} \text{c(A)} = 30 & \text{c(AB)} = 60 & \\ \text{c(B)} = 40 & \text{c(AC)} = 55 & \text{c(ABC)} = 80 \\ \text{c(C)} = 50 & \text{c(BC)} = 70 & \end{array}$$

   The payments of players A, B, C under the Shapley value is:

   (a) (15; 30; 35)
   (b) (17.5; 30; 32.5)
   (c) (17.5; 27.5; 35)
   (d) (20; 30; 30)
   (e) (22.5; 27.5; 30)

4. Which of the following methods is (are) consistent?
   (a) Point system that is used to demobilize US soldiers at the end of WWII.
   (b) Talmudic solution
   (c) Webster's rule
   (d) All of the above
   (e) None of the above

5. Consider the following cost sharing problem. There are four players A, B, C, D with the following costs:

c(A) = 6	c(AB) = 10	c(ABC) = 14	c(ABCD) = 20
c(B) = 7	c(AC) = 13	c(ABD) = 13	
c(C) = 7	c(AD) = 12	c(ACD) = 18	
c(D) = 6	c(BC) = 13	c(BCD) = 15	
	c(BD) = 12		
	c(CD) = 13		

   Consider a cost allocation that is in the core. Which of the following statements concerning such an allocation is false?
   (a) A should pay exactly 6
   (b) C should pay exactly 7
   (c) D should pay exactly 6
   (d) B & C together should not pay more than 13
   (e) B should not pay less than 2

6. Consider the following claims problem: There are four individuals and a total of \$875 to divide between them. Claims are as follows: $c_1 = 200$, $c_2 = 250$, $c_3 = 400$, $c_4 = 550$. What is the solutions to this claims problem suggested by the Talmudic solution?
   (a) (100; 125; 250; 400)
   (b) (125; 125; 275; 350)
   (c) (125; 125; 300; 325)
   (d) (200; 200; 200; 275)
   (e) (200; 225; 225; 225)

7. Which of the following apportionment methods is unbiased?
   (a) Adams'
   (b) Jefferson's

(c) Hamilton's
(d) Hill's
(e) None of the above

8. Consider the following claims problem: There are three individuals and a total of \$650 to divide between them. Claims are as follows: $c_1 = 200$, $c_2 = 300$, $c_3 = 500$. What is the Shapley value?

   (a) (100; 150; 400)
   (b) (125; 175; 350)
   (c) (125; 200; 325)
   (d) (150; 200; 300)
   (e) (150; 175; 325)

Short Answer Questions

9. There are two claimants with claims c_1, c_2 and the total amount available is a_0 (here $a_0 \leq c_1 + c_2$). How does the contested garment rule allocate the total amount between the two claimants?

10. In a claims problem there are two important cases to distinguish. What are these cases and why are they important to distinguish?

11. What is the relationship between the decomposition principle and the Shapley value in the context of cost allocation?

12. Explain the normative theories of justice by Aristotle, Bentham, and Rawls. What are their weaknesses?

Problem

13. Consider the following apportionment problem: There are a total of 25 seats to be allocated. There are five states with the following populations: State A (13), State B (26), State C (39), State D (74), and State E (98). Find the allocations suggested by Hamilton's, Jefferson's, Webster's, Adam's, and Hill's methods. (Please show your work.)